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A GAME THEORETIC APPROACH TO COORDINATION OF PRICING, ADVERTISING, AND INVENTORY DECISIONS IN A COMPETITIVE SUPPLY CHAIN

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Abstract. Supply chain members coordinate with each other in order to obtain more profit. The major mechanisms for coordination among supply chain echelons are pricing, inventory management, and ordering decisions. Regarding to these mechanisms, supply chain participants have conflicts of interest. This paper concerns coordination of enterprise decisions including pricing, advertising, ordering, and inventory decisions in a multi-echelon supply chain consisting of multiple suppliers, one manufacturer, and multiple retailers. In the current study, a novel inventory model is presented for both the manufacturer, and the retailers who are able to determine the number of orders for each product. Moreover, each supply chain member has equal power and make their decisions simultaneously. The proposed model considers the relationships among three echelon supply chain members based on a non-cooperative Nash game with pricing and inventory decisions. An iterative solution algorithm is proposed to find Nash equilibrium point of the game. Several numerical examples are presented to study the application of the model as well as the effectiveness of the algorithm. Finally, a comprehensive sensitivity analysis is performed and some important managerial insights are highlighted.

1. Introduction.

A supply chain is the cumulative effort of multiple firms which leads to deliver final products to end customer. Generally, in the supply chain, suppliers supply raw materials, manufacturer produce products, and retailers sell the final products to the end customers. Increasing competition due to market globalization, stimulates independent firms to coordinate in the supply chain which allows them to gain more mutual benefit. Pricing, advertising, inventory, and ordering decisions, the most important coordination mechanisms in the supply chain, are applied to improve profit of both the supply chain and individual firms ([36], [39]).

A supply chain often provides more than one type of products in order to response different customer needs. Thus, the supply chain may charge various prices for different products by retailers, also each product may have its advertising expenditure.

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Therefore, decreasing the price of each product and increasing the advertising expenditure lead to increasing demand and vice versa. The research in this paper has been considered multi products supply chain into the pricing, advertising, ordering, and inventory decisions to coordinate a decentralized supply chain.

In this paper, we investigate a supply chain with multiple suppliers, one manufacturer, and multiple retailers who are involved in supplying raw materials, producing, and selling multiple finished products, respectively. The manufacturer purchases multiple components or raw materials according to their needs from multiple suppliers and produces the finished products limited by common cycle interval approach and wholesales them to its retailers. This supply chain therefore has three echelon of members, the suppliers of raw materials, the manufacturer, and the retailers (e.g. [16]).

The main decisions of the suppliers are the prices of the raw materials, and quantity of raw materials. The manufacturer determines common production interval, wholesale prices, and the required amount of raw materials to optimize his net profit. Finally, the retailers buy each product from the manufacturer with specific replenishment cycle of own product at the wholesale prices, and then sell them to their end customers market at the retail prices. The markets of retailers are assumed to be geographically dispersed and independent of each other. The demand rate in each retail market is assumed to be an increasing function of the advertising expenditures made by the corresponding retailer, and a decreasing function of the retail price.

We present our mathematical model as a three-echelon non-cooperative game in which each supply chain member is assumed as a player. The suppliers compete at the bottom-level with each other. Simultaneously, they play with the manufacturer in another game. Also the retailers formulate non-cooperative game and at the same time compete with the manufacturer as a whole game, too. We have the Nash game, because we assume that all players determine their strategies simultaneously, and they have equal power in the market. Accordingly, an equilibrium solution of whole game is a solution in which no player has anything to gain by changing only its own strategy. In other words, if each player has chosen a strategy and no player can benefit by changing his or her strategy while the other players keep theirs unchanged, then the current set of strategies is a Nash equilibrium. Finally, we present analytical and computational methods to obtain the Nash equilibrium solution of the proposed game.

The remains of this paper is organized as follows. In section 2, the related researches are briefly reviewed. The assumptions and the notations used for the mathematical model are in section 3. Section 4 describes the analytical and computational methods for solving the models. In section 5, some numerical example are reported in order to discuss rewarding managerial insights. Eventually, concluding remarks are included in section 6.

2. **Literature review.**

Coordination can improve the performance of a supply chain such as low inventory cost, suitable utilization of production capacity, high customer satisfactory ([27]). Thereby, several researches have studied channel coordination for multi-echelon supply chain. However there are a few studies which considered pricing, and advertising of coordination for more than two echelon supply chain. A comprehensive review on channel coordination has been studied by [14].

2.1. **Two echelon supply chain.** Many researchers investigate pricing and inventory coordination in supply chains. [22], and [1] prove that the retailer(s) could achieve more profits when coordinating the price and order quantity decisions in various environments. [38], [31] [12], and [32] consider joint pricing and order inventory problem for profit optimization when price dependent demand in two echelon supply chain. They show that the supply chain members can obtain more profit in comparison with the situation in which they make decisions individually. [29] considered coordination of lead-time, order dependent purchasing cost, order size, reorder point, and lead time dependent partial backlogging in stochastic environment. [15] developed an economic order quantity model for on-instantaneous products with selling price under the impact of inflation and customer returns.

Multiple products supply chain is more complicated due to limited production capacity and customer needs ([20]). [44], and [10] studied a two-product joint pricing and inventory management problem. Also substitution effect of products is included in their model to improve its profit rather than two products study independently.

Advertising in supply chain is a form of marketing expenditure used to motivate end customers in order to grow their demand. Cooperative advertising is a formal contract, where a manufacturer offers to share a specific percentage of his retailer's advertising expenditures. Many researchers including [18], [19], and [42] proposed a static model for one retailer. [2] proposed a vertical cooperative advertising model for duopolistic retailers with a single manufacturer.

Demand function is another main criterion of mathematical formulation. Concerning the mathematical modeling of advertising costs, it is obvious that a linear integration is generally used in terms of advertising expenditures ([34], and [37]), while non-linear advertising expenditures are rare ([11], [25], and [41]). These articles studied cooperative advertising in a two-echelon supply chain, as well.

The above mentioned researches investigated coordination of pricing, advertising, and inventory management in a two-echelon supply chain channel including manufacturer(s) and retailer(s).

2.2. **Multi-echelon supply chain.** There are several researches considered coordination of inventory, and production decisions in a multi-echelon supply chain. [28] proposed an integrated production-inventory model for a three-echelon supply chain and determined optimal order size of raw materials, production rate and unit production cost, and idle times. [16] presented a new model for a multi-echelon supply chain which both pricing and inventory decisions have been determined in each echelon. [30] investigated production lot size problem for perfect and imperfect products in a three layer supply chain.

Based on [3] review, the superior supply chain component is a bilateral monopoly consisting of one manufacturer and one retailer while only few papers study the interaction of more than two players. [8] proposed a three-echelon model for a pricing game and [17] presented mathematical models for three contract mechanisms. Also [3] mentioned that multi-echelon models combined with the analysis of more than two decision variables e.g., advertising, pricing, and quality could be a promising research area. Thereby, we study a three-echelon supply chain in which each player makes pricing decisions and competes with others simultaneously. Also advertising, and ordering decisions are determined by the retailers, and the common production interval is the main decision of the manufacturer.

2.3. **Nash equilibrium for supply chain games.** Supply chain and inventory decisions are often keen on devising game mechanisms to improve the members' profits of studied supply chains where supply chains have two inventory echelons (one is for the manufacturer, and the second is for the retailers), and inventory policies are mainly based on classic EOQ models ([26]). Nash equilibrium for analyzing the game in supply chains are presented in many articles to show the independence of supply chain members, and improving the profits.

[9] proposed models by assuming which of the customers' demands are sensitive to the price and service quality. Nash equilibrium is also used by [6] to study the impact of price discount contracts in different scenarios and compared with some other games. They demonstrate that the price discount contracts outperform compared to the non-contract scenarios. [43] considered pricing decisions for two substitutable products in a two-echelon supply chain with two competitive manufacturers, and one common retailer. They developed one centralized pricing and seven decentralized pricing models. [40] investigated a multi-echelon supply chain where interactions among supply chain members were captured under the generalized Nash equilibrium assumption.

In this paper, we formulate a three-echelon supply chain consisting of multiple suppliers, one manufacturer, and multiple retailers. All supply chain members make pricing decisions including raw material prices, wholesale prices and retail prices, and they compete with each other, simultaneously. Moreover, advertising, ordering, and inventory decisions are included for the supply chain. Computational and analytical methods present to solve the Nash equilibrium of the whole game of supply chain.

3. **Problem description.** We present a three-echelon supply chain consisting of multiple suppliers, single manufacturer, and multiple retailers, where suppliers sell multiple raw materials to the manufacturer. The manufacturer produces multiple products and wholesales them to the retailers, who receive the products in a limited number of orders determined by retailers and finally sell the products to end customers. Figure 1 shows relations among the supply chain members.

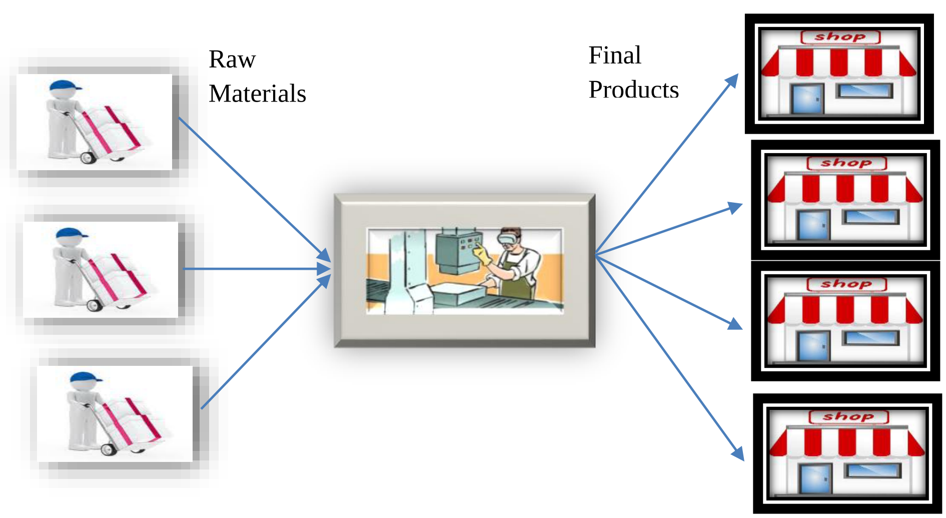

FIGURE 1. Scheme of a three-echelon supply chain

3.1. **Notation and assumptions.** Following assumptions are considered for the proposed model of this paper:

1. The number of orders for each product of the retailers is positive integer, and is determined by retailers ([23], [7], and [16]
2. There are one manufacturer and multiple retailers. The manufacturer's production capacity for multiple products is limited. In addition, the suppliers' capacities for providing raw materials are restricted. The retailers are independent from each other and have their own individual markets. ([12], and [25]).
3. Shortage is not permitted. Therefore, the total production rate of multiple products is greater than demand rate [11].
4. All parameters of supply chain members are deterministic and known in advance ([39]).
5. For each raw material of each product which manufacturer produces and wholesales to retailers, at least two suppliers are available. In other words, due to considering competition among suppliers, single sourcing is not permitted ([33]).

The relevant parameters and decision variables of supply chain participants are shown in Table 1, and Table 2, respectively.

TABLE 1. Parameters of suppliers, manufacturer, and retailers

I	Index of product number ($i = 1, 2, ..., n$)
R	Index of retailers ($r = 1, 2, ..., R$)
S	Index of suppliers ($s = 1, 2, ..., S$)
J	Set of raw materials needing for producing n products
f_{ir}	Scaling constant for demand function ($f_{ir} > 0$)
h_{ir}	Retailer r's holding cost per unit of i^{th} product
Ar_{ir}	Retailer r's ordering cost of i^{th} product
α_{ir}	Retailer r's price elasticity of demand function for product i
β_{ir}	Retailer r's advertising expenditure elasticity of demand function for product i ($\beta_{ir} > 0, \alpha_{ir} > 1 + \beta_{ir}$)
P_i	Annual production capacity for i^{th} product
Cm_i	Manufacturer's production cost per unit of product i
As_i	Manufacturer's setup cost of product i
hm_i	Manufacturer's holding cost per unit for product i
B_m	Maximum production budget of all products
u_{ji}	Predefined usage amount of unit j^{th} raw material per unit product i
Cs_{js}	The production cost of j^{th} raw material paid by supplier s
Ca_{js}	Maximum production capacity of j^{th} raw material for supplier s
η_{js}	Supplier s's self-price elasticity for raw material j
θ_{js}	Supplier s's competitors-price elasticity for raw material j

3.2. **The retailers' model formulation.** The demand of each product at retailer r $D(p_{ir}, a_{ir})$ is a joint non-linear function of the retail prices and advertising expenditures in real world. It is decreasing with regard to retailing price and increasing with regard to advertising expenditure. Demand's Equation is as follows ([11], and [25]):

$$D\left(p_{ir}\ ,\ a_{ir}\right) = \ f_{ir}\ p_{ir}^{-\alpha_{ir}}\ a_{ir}^{\beta_{ir}} \tag{1}$$

TABLE 2. Decision variables of suppliers, manufacturer, and retailers

k_{ir}	Number of orders of retailer r for product i (positive integer variable)
p_{ir}	Retailing price of r^{th} retailer for product i
a_{ir}	Advertising expenditure of r^{th} retailer for product i
T	Common production interval
ψ_i	The unit wholesale price of product i
Q_j	Amount required of j^{th} raw material to produce all products
F_{js}	The price of j^{th} raw material charged by the supplier s to the manufacturer
v_{js}	The amount of j^{th} raw material produced by supplier s

Where f_{ir} is a positive scaling parameter. α_{ir} and β_{ir} are the price elasticity and advertising expenditure elasticity of each product, respectively.

Each retailer's main objective is to maximize his net profit by optimizing his decision variables including number of orders, retail price, and advertising expenditure. These set of decision variables are known strategy X_{R_r}. Thus, the retailer r's net profit can be calculated as the total sales revenue of multiple products minus the purchasing cost from the manufacturer, advertising cost for each product, ordering and holding costs with number of orders, given as follows.

$$\begin{aligned}\max_{p_{ir},a_{ir},k_{ir}} \Pi R_r = &\sum_{i=1}^{n} p_{ir}D_{ir} - \sum_{i=1}^{n} \psi_{ir}D_{ir} - \sum_{i=1}^{n} a_{ir}D_{ir} \\ &- \frac{T}{2}\sum_{i=1}^{n}\frac{D_{ir}h_{ir}}{k_{ir}} - \sum_{i=1}^{n}\frac{Ar_{ir}k_{ir}}{T}\end{aligned} \tag{2}$$

$$\text{s.t. } k_{ir} \in \{1,2,3,...\} \tag{3}$$

$$a_{ir}, p_{ir} \geq 0 \tag{4}$$

Based on first assumption of the model, a positive integer k_{ir} defined as the number of orders which is employed between the manufacturer and the retailers for each product. The common production interval of the manufacturer in which all the products are forced to be produced denoted by T. Therefore, the replenishment cycle for retailer r can be calculated as $\frac{T}{k_{ir}}$. The annual holding cost and the ordering cost is fourth and fifth terms of objective function (see Figure 2).

Constraint (3) shows the number of orders of retailer r for each product which must be a positive integer. Constraint (4) ensures that the value of retailing price and advertising expenditure of each product are nonnegative.

3.3. **The manufacturer's model formulation.** Consider the inventory level of the manufacturer system first. Figure 3, shows the behavior of inventory level for the two products of the manufacturer. The average annual inventory for i^{th} product through algebraic manipulation (refer to [21], and [16]) is obtained from Equation (5). Also shortage is not permitted for the manufacturer, the production rate must be at least equal to the expected demand rate ($P_i \geq \sum_{r=1}^{R} D_{ir}$). In such multiproduct manufacturing system, if $\sum_{i=1}^{n}\frac{\sum_{r=1}^{R} D_{ir}}{P_i} \leq 1$, then is possible a feasible time interval, T.

$$\overline{I} = \frac{T}{2}\sum_{i=1}^{n}\sum_{r=1}^{R} D_{ir}\left(1 + \frac{1}{k_{ir}} - \frac{\sum_{r=1}^{R} D_{ir}}{P_i}\right) \tag{5}$$

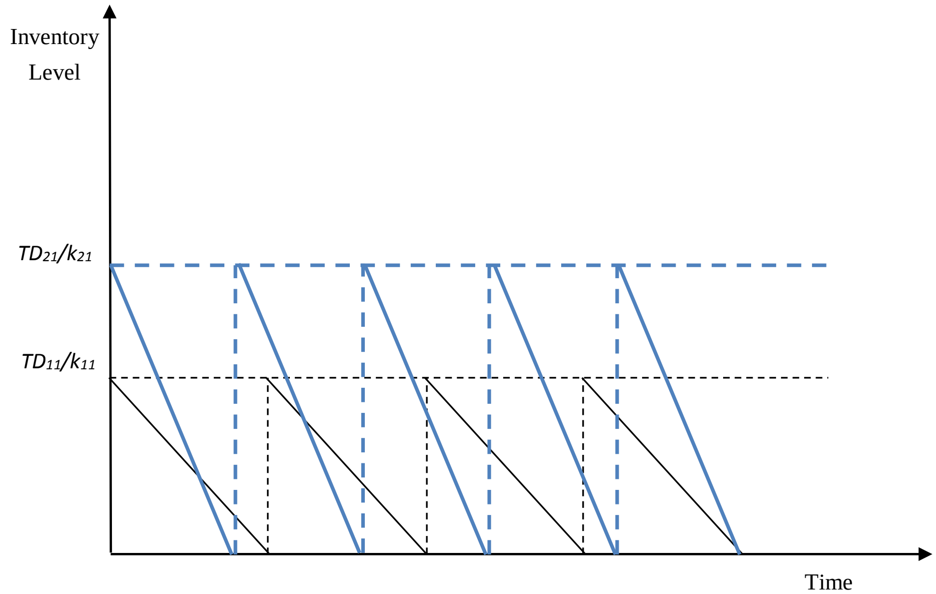

FIGURE 2. Inventory level of two final products for retailer 1

We assume that the unit wholesale price ψ_i of each product i, the common production interval T, and the amount of required raw materials Q_j are the manufacturer's decisions. Then, the strategy of manufacturer is denoted by X_m. The manufacturer's net profit equals to the wholesales revenue of multiple products to all retailers minus the production cost of each product needed by the retailers, purchasing cost of raw materials from the whole suppliers, and setup and holding costs. Thus, the manufacturer's objective function is shown as follows.

$$\begin{aligned}\max_{\psi_i,T,Q_j} \Pi_M = \sum_{i=1}^{n} \psi_i \sum_{r=1}^{R} D_{ir} - \sum_{i=1}^{n} Cm_i \sum_{r=1}^{R} D_{ir} - \sum_{j=1}^{J} \sum_{s=1}^{S} F_{js} v_{js} \\ - \frac{\sum_{i=1}^{n} As_i}{T} - \frac{T}{2} \sum_{i=1}^{n} hm_i \sum_{r=1}^{R} D_{ir} \left(1 + \frac{1}{k_{ir}} - \frac{\sum_{r=1}^{R} D_{ir}}{P_i}\right)\end{aligned} \tag{6}$$

$$\text{s.t. } T \sum_{i=1}^{n} Cm_i \sum_{r=1}^{R} D_{ir} \leq B_m \tag{7}$$

$$Q_j = u_{ji} \sum_{r=1}^{R} D_{ir} \qquad \text{for } j = 1, 2, ..., J \tag{8}$$

$$\psi_i \geq 0, T > 0, Q_j \geq 0 \tag{9}$$

Constraint (7) shows that the manufacturer can spend at most Bm units of budget for production of all products. In other words, manufacturer cannot produce all products as much as he wants. Constraint (8) ensures that the amount of required raw materials are satisfied such that all demands of retailers are covered. Thus, there is no shortage in the supply chain. The value range of decision variables are shown by constraint (9).

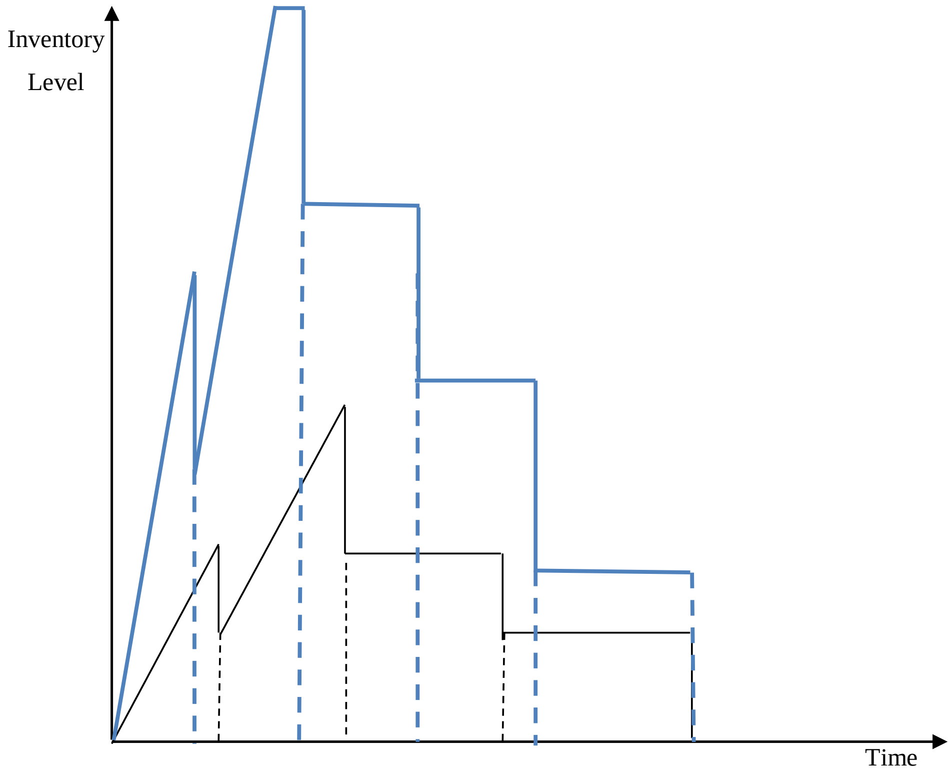

Figure 3. Inventory level of two products for manufacturer

3.4. **The suppliers' model formulation.** Each supplier's decision variables are pricing decisions for the raw materials, and the amount of raw materials. The strategy X_{S_s} shows these set of decision variables. Thus, suppliers must obtain their optimal strategies in order to maximize their net profits. The supplier s faces production cost of raw materials for manufacturer, and his revenue can be achieved by selling the raw materials to manufacturer. Therefore, the supplier s objective function is as follows.

$$\max_{F_{js}, v_{js}} \Pi_{S_s} = \sum_{j=1}^{J} F_{js} v_{js} - \sum_{j=1}^{J} Cs_{js} v_{js} \tag{10}$$

$$\text{s.t. } v_{js} = Q_j - \eta_{js} F_{js} + \sum_{s=1/s}^{S} \theta_{js} F_{js} \tag{11}$$

$$\sum_{s=1}^{S} v_{js} = Q_j \qquad \text{for } j = 1, 2, ..., J \tag{12}$$

$$v_{js} \le Ca_{js} \qquad \text{for } j = 1, 2, ..., J \tag{13}$$

$$v_{js} \ge 0, F_{js} \ge 0 \tag{14}$$

Equation (11) shows that the demand of each raw material depends on self-offered price of raw materials as well as other-offered price. In other words, in order to gain more market share, not only suppliers compete with manufacturer, but also they compete with each other. Equation (12) ensures amount of required raw materials supplied by suppliers, and this constraint is common for all suppliers that can produce j^{th} raw material. Thereby, in suppliers' level we face i^{th} a Generalized Nash Equilibrium Point (GNEP). Production capacity of each supplier for each raw material is considered in constraint (13). Finally, constraint (14) ensures non-negativeness of decision variables.

4. **Solution method.** We consider the above formulation for a three-echelon supply chain using non-cooperative game theory approach with $S + 1 + R$ players in which there are S suppliers, one manufacturer, and R retailers. In this research, in order to analyze the strategies of suppliers, manufacturer, and retailers, we apply Nash equilibrium concept. In this section, at first we describe game theory approach, then we obtain best response of each player. Finally, solution procedure is proposed to achieve Nash equilibrium.

4.1. **Game theory approach.** Each supply chain member (player) controls his strategy set. As it is mentioned in previous section, each supplier decides on his strategy set $X_{S_s} (\forall s \in S)$ to maximize his objective function. A strategy $x_{S_s} \in X_{S_s}$ consists of pricing and production amount decisions for each raw material provided by a supplier. The manufacturer determines his strategy set X_m to maximize his net profit function. A strategy $x_m \in X_m$ includes common production cycle, wholesale price, and required amount of raw materials. The strategy set of each retailer is shown by X_{R_r} to maximize his payoff function. His strategy $x_{R_r} \in X_{R_r}$ consists of number of orders, retailing price, and advertising expenditure decisions.

We consider the suppliers as a bottom-level in which they compete simultaneously with each other as well as with manufacturer. In order to gain a greater market share, the suppliers compete with each other. Moreover, to maximize their profit, they compete with manufacturer. In other words, in our game framework we face with horizontal and vertical competition in bottom-level. At the same time, retailers compete with manufacturer as non-cooperative game. We employ Nash equilibrium concept to analyze the whole model (all players, i.e. suppliers, manufacturer, and retailers).

In game theory, Nash equilibrium is one of the most famous non-cooperative solution concept [4]. To achieve Nash equilibrium point, each player determine his own strategy according to the other player's strategies as given input parameters and adapt his decision corresponding to the changing of the other player's strategies to maximize his objective function. This process proceeds until any players is not desiring to change his strategy unilaterally, because any one-sided changing lead to loss to him and then Nash equilibrium point is obtained.

4.2. **Best response of players.** In this section we investigate the best responses of retailers, manufacturer, and suppliers according to given input parameters of other players.

4.2.1. *Best responses of retailers.* To obtain optimal decisions of retailers, we assume the manufacturer's strategies are known. At first we show the objective function of retailers is strictly pseudo concave.

Lemma 4.1. *Objective function of retailers is pseudo concave with respect to p for fixed a.*

Proof. See Appendix A(i). □

Lemma 4.2. *Objective function of retailers is concave with respect to a_{ir}, k_{ir}.*

Proof. See Appendix A (ii). □

According to Lemma 2 and since k_{ir} is positive integer and the objective function is concave, based on [35] the optimal k_{ir} is obtained as follows:

$$k_{ir}^{*}=\left\lfloor\frac{\left(1+\sqrt{1+\frac{2T^{2}D_{ir}h_{ir}}{Ar_{ir}}}\right)}{2}\right\rfloor \tag{15}$$

Corresponding to Lemma 1 and 2, and first-order condition of Equation (2) yields

$$a_{ir}^{*}=\frac{\beta_{ir}\left(p_{ir}-\psi_{i}-\frac{Th_{ir}}{2k_{ir}}\right)}{\beta_{ir}+1} \tag{16}$$

$$p_{ir}^{*}=\frac{a_{ir}\left(a_{ir}+\psi_{i}+\frac{Th_{ir}}{2k_{ir}}\right)}{a_{ir}-1} \tag{17}$$

Substituting (17) to (16) gives

$$a_{ir}^{*}=\frac{\beta_{ir}\left(\psi_{i}+\frac{Th_{ir}}{2k_{ir}}\right)}{a_{ir}-\beta_{ir}-1} \tag{18}$$

By substituting (18) into (17), we obtain p^{*}.

$$p_{ir}^{*}=\frac{a_{ir}\left(\psi_{i}+\frac{Th_{ir}}{2k_{ir}}\right)}{a_{ir}-\beta_{ir}-1} \tag{19}$$

Therefore Equations (15), (18), and (19) are the best responses of each retailer.

4.2.2. *Best responses of manufacturer.* Suppose that the strategies of suppliers and retailers are fixed. We prove the objective function of manufacturer is concave with respect to his decision variables.

Lemma 4.3. *Objective function of manufacturer is concave respect to ψ_i, and T.*

Proof. See Appendix A(iii). □

Also constraints of manufacturer are linear, so constraint set of manufacturer is convex. To obtain optimal decision variables, we use KKT conditions. Thereby, Lagrange equation and first order condition of manufacturer's objective function are as follows (Equations (20) to (24)).

$$\begin{aligned}L_{M}(\psi_{i},T,Q_{j})=\Pi_{M}(\psi_{i},T,Q_{j})-\lambda.g(T)&=\sum_{i=1}^{n}\psi_{i}\sum_{r=1}^{R}D_{ir}-\sum_{i=1}^{n}Cm_{i}\sum_{r=1}^{R}D_{ir}\\ -\sum_{j=1}^{J}\sum_{s=1}^{S}F_{js}v_{js}-\frac{\sum_{i=1}^{n}As_{i}}{T}-\frac{T}{2}\sum_{i=1}^{n}hm_{i}\sum_{r=1}^{R}D_{ir}&\left(1+\frac{1}{k_{ir}}-\frac{\sum_{r=1}^{R}D_{ir}}{P_{i}}\right)\\ &-\lambda T\left(\sum_{i=1}^{n}Cm_{i}\sum_{r=1}^{R}D_{ir}-B_{m}\right)\end{aligned} \tag{20}$$

$$\nabla L_{m}(\psi_{i},T,Q_{j})=0\rightarrow\nabla\Pi_{m}(\psi_{i},T,Q_{j})-\lambda.\nabla g(T)=0 \tag{21}$$

$$\lambda.\nabla g(T) = 0 \tag{22}$$

$$\begin{bmatrix} \frac{\partial \Pi_M(\psi_i,T,Q_j)}{\partial \psi_{ir}} \\ \frac{\partial \Pi_M(\psi_i,T,Q_j)}{\partial T} \end{bmatrix} - \lambda \begin{bmatrix} \frac{\partial g(T)}{\partial \psi_{ir}} \\ \frac{\partial g(T)}{\partial T} \end{bmatrix} = 0 \tag{23}$$

$$\begin{bmatrix} D_{ir} \\ \frac{\sum_{i=1}^{n} As_i}{T^2} - 0.5\sum_{i=1}^{n} hm_i \sum_{r=1}^{R} D_{ir}\left(1+\frac{1}{k_{ir}}-\frac{\sum_{r=1}^{R} D_{ir}}{P_i}\right) - \lambda\left(\sum_{i=1}^{n} Cm_i \sum_{r=1}^{R} D_{ir}\right) \end{bmatrix} = 0 \tag{24}$$

The optimal common production interval is as follows:

$$T^* = \sqrt{\frac{\sum_{i=1}^{n} As_i}{0.5\sum_{i=1}^{n} hm_i \sum_{r=1}^{R} D_{ir}\left(1+\frac{1}{k_{ir}}-\frac{\sum_{r=1}^{R} D_{ir}}{P_i}\right) + \lambda\left(\sum_{i=1}^{n} Cm_i \sum_{r=1}^{R} D_{ir}\right)}} \tag{25}$$

Where optimal λ according to Equation (22), is as follows:

$$\lambda^* = \max(0, \frac{(\sum_{i=1}^{n} As_i).(\sum_{i=1}^{n} Cm_i \sum_{r=1}^{R} D_{ir})^2 - (B_m)^2.(0.5\sum_{i=1}^{n} hm_i \sum_{r=1}^{R} D_{ir}\left(1+\frac{1}{k_{ir}}-\frac{\sum_{r=1}^{R} D_{ir}}{P_i}\right))}{(\sum_{i=1}^{n} Cm_i \sum_{r=1}^{R} D_{ir}).(B_m)^2}) \tag{26}$$

Solving $\Pi_M(\psi_i) = 0$ for zero profit gives:

$$\psi_i^0 = Cm_i + \frac{1}{\sum_{r=1}^{R} D_{ir}}\left(\frac{\sum_{i=1}^{n} As_i}{T} + \frac{T}{2}.\sum_{i=1}^{n} hm_i \sum_{r=1}^{R} D_{ir}\left(1+\frac{1}{k_{ir}}-\frac{\sum_{r=1}^{R} D_{ir}}{P_{ir}}\right) + \sum_{j=1}^{J}\sum_{s=1}^{S} F_{js}v_{js}\right) \tag{27}$$

Since (27) is an increasing linear function of ψ_i, the optimal ψ_i occurs at the highest price it is possible for the manufacturer to charge the retailers for each product. Thus, for some $C_i > 1$,

$$\psi_i^* = C_i\psi_i^0 = C_i\left(Cm_i + \frac{1}{\sum_{r=1}^{R} D_{ir}}\left(\frac{\sum_{i=1}^{n} As_i}{T} + \frac{T}{2}.\sum_{i=1}^{n} hm_i \sum_{r=1}^{R} D_{ir}\left(1+\frac{1}{k_{ir}}-\frac{\sum_{r=1}^{R} D_{ir}}{P_{ir}}\right) + \sum_{j=1}^{J}\sum_{s=1}^{S} F_{js}v_{js}\right)\right) \tag{28}$$

Also amount of required raw materials (Q_j) are obtained from Equation (8). Hence, the best response of manufacturer is achieved.

4.2.3. *Best responses of suppliers.* As mentioned in Section 3, suppliers' strategies are composed of price and production amount of raw materials. To obtain best responses of suppliers, assume that the strategies of manufacturer and retailers are fixed as well. We need to prove suppliers' objective function are concave with respect to their decision variables.

Lemma 4.4. *Objective function of suppliers are concave respect to F_{js}, v_{js}.*

Proof. See Appendix A (iv). □

To obtain the optimal the best response, Constraint (13) is not considered initially. Thus, Lagrange equation and first order condition of suppliers' objective function is as follows.

$$\begin{aligned} L_{S_s}(F_{js}) = & \sum_{j=1}^{J}\sum_{s=1}^{S} F_{js}\left(Q_j - \eta_{js}F_{js} + \sum_{s=1/s}^{S}\theta_{j\underline{s}}F_{j\underline{s}}\right) \\ & - \sum_{j=1}^{J}\sum_{s=1}^{S} Cs_{js}\left(Q_j - \eta_{js}F_{js} + \sum_{s=1/s}^{S}\theta_{j\underline{s}}F_{j\underline{s}}\right) - \mu_j\left(\sum_{s=1}^{S} v_{js} - Q_j\right) \end{aligned} \tag{29}$$

$$\begin{aligned} \frac{\partial L_{S_s}(F_{js})}{\partial F_{js}} = 0 \rightarrow & \\ & \left(Q_j - \eta_{js}F_{js} + \sum_{s=1/s}^{S}\theta_{j\underline{s}}F_{j\underline{s}}\right) - \eta_{js}(F_{js} - Cs_{js}) \\ & - \mu_j(-\eta_{js} + (s-1)\theta_{js}) = 0 \end{aligned} \tag{30}$$

For each supplier that produced j^{th} raw material, there are an Equation (30). In addition, Equation (12) is satisfied. Solving simultaneously mentioned linear equations that can easily be done, optimal decision variables of supplier are achieved. It is worth noting, μ_j might be either positive or negative. If the solution set of equations is satisfied in Constraint (13), optimal decision variables are obtained. Otherwise, $v_{js} = Ca_{js}$ and set of equations resolved as long as all v_{js} is satisfied in Constraint (13). Therefore, the best responses of suppliers are obtained.

4.3. **Algorithm procedure.** We now propose a solution procedure based on Gauss-Seidel decomposition presented by [13] to calculate the Nash equilibrium of three-echelon competitive supply chain. As mentioned in previous section, X_{R_r}, X_m, X_{S_s} are the strategy sets of retailers, manufacturer, and suppliers, respectively. We give the following algorithm procedure to find Nash equilibrium of proposed non-cooperative game model:

Step 1. Give the initial solution for all players $x^0 = (x^0_{Ss}, x^0_M, x^0_{Rr})$ where it is feasible for all supply chain members.

Step 2. In retailers' model, assume strategies of manufacturer, and suppliers are fixed, then the optimal responses of retailers $x^*_{Rr} = (p^*_{ir}, a^*_{ir}, k^*_{ir})$ are obtained as section 4.2.1.

Step 3. In manufacturer's level, suppose the strategies of retailers and suppliers respectively achieved from Step 2, and Step 1, are fixed. Then, calculate the best response of manufacturer $x^*_M = (\psi^*_{ir}, T^*, q^*_j)$ based on Section 4.2.2.

Step 4. Calculate the best responses of suppliers $x^*_{Ss} = (F^*_{js}, v^*_{js})$ as mentioned in Section 4.2.3 based on optimal strategy of manufacture, and retailers obtained from Step 3, and Step 2, respectively.

Step 5. If $\parallel x^*_{Rr} - x^0_{Rr} \parallel \leq \epsilon$, $\parallel x^*_M - x^0_M \parallel \leq \epsilon$, and $\parallel x^*_{Ss} - x^0_{Ss} \parallel \leq \epsilon$, the Nash equilibrium of proposed game is obtained. Otherwise, $x^0_{Rr} = x^*_{Rr}$, $x^0_M = x^*_M$, and $x^0_{Ss} = x^*_{Ss}$, go to Step 2. (ϵ is very small positive number).

5. **Numerical study and managerial insights.** In this section, we describe the application of the proposed game model and effectiveness of the proposed solution approach for a three-echelon competitive supply chain. We study several numerical

examples aimed at illustrating some important features of the model and managerial highlights.

Example 1. We exhibit a simple example to do a comprehensive sensitivity analysis. Suppose there is a supply chain including two suppliers $s = 1, 2$, one manufacturer, and one retailer $r = 1$ with two raw materials $j = 1, 2$ and two products $i = 1, 2$. Consider $f_{ir} = 10^5 * [9.5, 8]$, $\alpha_{ir} = [1.8, 1.5]$, $\beta_{ir} = [0.3, 0.1]$, $Ar_{ir} = [20, 15]$, and $hr_{ir} = [2, 4]$ for the retailer, and for the manufacturer $Cm_i = [2, 3]$, $hm_i = [7, 5]$, $As_i = [140, 180]$, $P_i = 10^3 * [10, 9]$, $C_i = [2, 2]$, $u_{ji} = [1, 0.05; 0.6, 0.2]$. Finally, parameters of two suppliers are $\eta_{js} = [18, 17; 26, 15]$, $\theta_{js} = 10^(- 2) * [1, 7; 2, 5]$, and $Cs_{js} = [0.5, 0.3; 0.6, 0.25]$. The Nash game produces the following optimal values for the retailer's decision variables: $p^*_{ir} = [162.01, 75.76]$, $a^*_{ir} = [27.00, 10.18]$, and $k^*_{ir} = [1, 3]$. The decision variables of the manufacturer are $\psi^*_i = [44.70, 40.54]$, $T^* = 0.2998$, and $Q^*_j = [589.77, 241.44]$. Eventually, decision variables of two suppliers are $F^*_{js} = [16.48, 17.32; 4.72, 7.95]$, and $v^*_{js} = [294.27, 295.50; 119.10, 122.34]$. The corresponding retailer's, manufacturer's, and two suppliers' profits are $\Pi R^*_1 = 78429.9$, $\Pi^*_M = 17914.2$, $\Pi S^*_1 = 5194.4$, and $\Pi S^*_2 = 5970.9$, respectively.

In order to investigate the model, we do a sensitivity analysis for main parameters including α_{ir}, β_{ir}, C_i, and η_{js}. Table 3, and Table 4 show the mean prices and objective values of supply chain members. To examine the impact of retail price elasticity, we change all the retail prices elasticity in the interval $[0.8\alpha_{ir}, 1.2\alpha_{ir}]$ and show its impact on the mean retail prices and whole sale prices of the two products, and the mean prices of two raw materials, respectively. As it can be seen, by increasing α_{ir}, all supply chain members offer lower prices and lose profit to some extent. In addition, impact of advertising elasticity can be shown in Table 3 by changing β_{ir} in interval $[0.8\beta_{ir}, 2.5\beta_{ir}]$. Increasing β_{ir} results in higher prices and accordingly higher objective values for all supply chain participants.

We analyze influence of wholesale prices, and raw materials' price elasticity in Table 4 by changing the related parameters in the intervals $[1.5C_i, 2.5C_i]$, and $[0.5\eta_{js}, 2.0\eta_{js}]$, respectively. The mean retail prices, and raw material prices increase by raising the wholesale prices. Besides, only the objective value of the manufacturer has increased, but the objective functions of suppliers and retailer have been reduced. In addition, Table 4 depicts that the mean price of two suppliers, the manufacturer, and the retailer can be reduced by raising price elasticity of raw materials and subsequently all the participants can benefit more due to this price reduction.

According to Table 3 one can infer that increasing self-price elasticity (α_{ir}) declines the mean prices of all the supply chain participants and consequently reduces the total benefits of the supply chain. In other words, the self-price elasticity of the retailers are high in much competitive markets where the players are forced to decrease their prices to capture more demand. The results in Table 3 justify this phenomenon. Contrarily, an increase in β_{ir} raises the mean prices of all the supply chain members and as a result they can earn more benefit. In fact, when the demand is much sensitive to advertising activities, the supply chain will be able to gain more benefit by spending on advertisement compared with the products with less sensitive demand.

According to Table 4, it is obvious that the higher coefficient of manufacturer unit cost results in higher wholesale prices and therefore the retailers and the suppliers will offer superior prices for the final products and raw materials respectively.

TABLE 3. Sensitivity analysis on price and advertising elasticity

Parameter	Coefficient	Mean Price				Objective Values			
		R	M	S_1	S_2	R	M	S_1	S_2
α_{ir}	0.8	591.54	52.85	13.76	16.52	4.43e+5	2.67e+4	8.74e+3	1.01e+4
	0.9	260.79	49.62	13.08	15.66	1.73e+5	2.48e+4	7.93e+3	9.11e+3
	1.1	110.28	37.08	8.19	9.70	3.93e+4	1.24e+4	3.08e+3	3.55e+3
	1.2	90.17	35.24	6.10	7.14	2.11e+4	8.49e+3	1.69e+3	1.96e+3
β_{ir}	0.8	143.20	42.08	10.47	12.46	7.25e+4	1.76e+4	5.08e+3	5.83e+3
	1.5	211.97	44.57	10.99	13.14	1.02e+5	1.89e+4	5.56e+3	6.41e+3
	2	324.84	45.92	11.32	13.55	1.51e+5	1.97e+4	5.88e+3	6.79e+3
	2.5	497.00	44.47	10.98	13.12	2.10e+5	1.89e+4	5.54e+3	6.39e+3

TABLE 4. Sensitivity analysis on coefficient of manufacturer unit cost and price elasticity of suppliers

Parameter	Coefficient	Mean Price				Objective Values			
		R	M	S_1	S_2	R	M	S_1	S_2
C_i	1.5	135.42	36.64	13.16	15.73	8.35e+4	1.32e+4	8.08e+3	9.25e+3
	1.75	146.58	39.68	11.74	14.01	8.08e+4	1.60e+4	6.40e+3	7.34e+3
	2.25	167.95	45.49	9.66	11.50	7.64e+4	1.92e+4	4.30e+3	4.95e+3
	2.5	178.38	48.33	8.87	10.54	7.45e+4	2.02e+4	3.60e+3	4.16e+3
η_{js}	0.5	203.86	55.23	14.61	17.42	7.02e+4	1.60e+4	4.99e+3	5.69e+3
	0.75	175.10	47.43	12.12	14.45	7.49e+4	1.71e+4	5.12e+3	5.86e+3
	1.5	135.69	36.73	8.76	10.42	8.36e+4	1.91e+4	5.26e+3	6.09e+3
	2.0	122.33	33.11	7.64	9.07	8.74e+4	2.01e+4	5.28e+3	6.14e+3

The manufacturer can achieve much more benefit in such situation, but both the suppliers and the retailers will gain lower benefit due to losing demand.

In the suppliers' echelon, each supplier tries to capture more demand from a competition with the other existing suppliers. The amount of raw materials required by the manufacture makes the potential market demand for respective suppliers. As it is shown in Table 4, the higher η_{js} is, the much competition among the suppliers will occur, which force them to offer lower prices to the manufacturer. As a result, the manufacturer and the retailers propose lower prices which leads to more demand obtained by the supply chain. Hence, all the supply chain participants benefit more due to this demand increment.

Example 2. Assume a supply chain consisting of $s = 1, 2, .., S$ suppliers, one manufacturer, and $r = 1, 2, , R$ retailers, where the manufacturer produces $i = 1, 2, , n$ products from $j = 1, 2, , J$ raw materials. All the parameters are generated randomly from uniform distributions summarized in Table 5. The size of the test problem is shown by $s \times r \times i \times j$.

TABLE 5. Parameters of the multi-echelon supply chain

Parameter	Range	Parameter	Range	Parameter	Range
f_{ir}	$10^5 * U(4, 10)$	α_{ir}	$U(1.6, 2.3)$	β_{ir}	$U(0.1, 0.7)$
Ar_{ir}	$U(10, 30)$	hm_i	$U(1, 6)$	Cm_i	$U(1, 10)$
hr_{ir}	$U(5, 20)$	As_i	$U(100, 500)$	P_i	$10^3 * U(8, 12)$
C_i	$U(1.5, 2.5)$	u_{ij}	$U(0.5, 1.5)$	η_{js}	$U(15, 30)$
θ_{js}	$U(0.01, 0.1)$	Cs_{js}	$U(0.1, 1)$		

We have generated several test problems with different sizes according to Table 5. All the instances have been solved using the proposed algorithm described in Section 4.3. The algorithm has converged to the Nash equilibrium point after a few number of iterations in a reasonable time.

TABLE 6. Results of the test problems

Test Problem	Mean Price			Mean Objective Values		
	R	M	S	R	M	S
$2 \times 2 \times 1 \times 3$	125.22	35.47	14.78	8.15e+4	1.47e+4	7.15e+3
$2 \times 2 \times 2 \times 3$	178.91	49.81	16.20	7.27e+4	1.25e+4	5.29e+3
$3 \times 2 \times 2 \times 4$	209.24	57.10	18.21	6.39e+4	1.02e+4	4.56e+3
$3 \times 2 \times 2 \times 5$	147.34	37.92	15.10	8.02e+4	1.32e+4	6.96e+3
$4 \times 2 \times 2 \times 4$	234.40	62.31	18.17	8.72e+4	1.79e+4	9.36e+3
$4 \times 3 \times 2 \times 5$	193.51	53.35	17.08	7.78e+4	1.33e+4	6.73e+3
$4 \times 3 \times 2 \times 6$	148.33	39.22	15.35	7.79e+4	1.51e+4	7.04e+3
$5 \times 3 \times 2 \times 7$	215.61	59.02	19.10	7.23e+4	1.04e+4	4.45e+3
$5 \times 2 \times 3 \times 8$	192.03	50.24	16.31	7.12e+4	1.15e+4	5.08e+3
$5 \times 2 \times 3 \times 10$	162.85	41.49	16.08	7.76+4	1.64e+4	7.83e+3

For each test problem, the mean price of raw materials for the suppliers, the mean wholesale price of all products for the manufacturer, the mean retail price of final products for the multiple retailers, and also the averages of the benefits in each echelon including the suppliers, the manufacturer, and the retailers have been summarized in Table 6. Furthermore, to validate the results we have applied a Branch and Bound algorithm as an alternative approach for solving the mixed integer non-linear sub problems of the retailers described in section 4.2.1. The Branch and Bound algorithm has been coded in GAMS 24.1.2 software solving by the SBB solver. The results show that both the proposed algorithm and the Branch and Bound algorithm have converged to the same Nash equilibrium point.

6. **Concluding remarks.** In this paper, we studied the coordination of pricing, advertising expenditure, ordering, and production decisions in a three-echelon competitive supply chain including multiple retailers, one manufacturer, and multiple suppliers. All supply chain participants make pricing decisions simultaneously. An inventory model have been developed in which both the manufacturer, and the retailers determined the number of orders for each product. These coordination decisions have been formulated as a non-cooperative game. The suppliers compete each other and with the manufacturer simultaneously. At the same time the retailers compete with the manufacturer. To obtain Nash equilibrium, an iterative solution algorithm has been proposed. In order to interrogate the proposed model

and solution algorithm some numerical examples have been provided and managerial implications have been drawn from the results.

The numerical examples show that if the retailers propose higher prices, then manufacturer, and respective suppliers sell their products with higher prices. In addition, price elasticity of retailers act as the main factor in the realized demand and profit of the whole supply chain. Since wholesale price of the manufacturer is influenced by his own prime cost, the less prime cost he endures, the less will be the product retailing price as well as the respective raw materials prices. Moreover, the suppliers with less self-price elasticity factor can propose higher prices to the manufacturer.

For future research, it would be interesting to extend the model to consider the competition for large size problems with more products and more retailers. Stackelberg game as the other non-cooperative game theory approach, and cooperative game are also worth to be addressed. In a multiple period environment, dynamic pricing with advertising expenditure, and inventory decisions will also be interesting areas to be studied.

Appendix A.

(i) [24] show his objective function of buyer is strictly pseudo concave. Simply we replace Equation (19) of their paper ($A = \psi + M + A_b/Q$), with $A = \psi_i + a_{ir} + \frac{(T.h_{ir})}{(2*k_{ir})}$. The rest of the proof is similar to their proof, and since our objective function is separable function, then objective function is pseudo concave and proof is completed. □

(ii) Second derivative of function ΠR_r based on [11]:

$$\frac{\partial^2 \Pi R_r(p_{ir}^*, a_{ir}^*)}{\partial a_{ir}^2} = \frac{\alpha_{ir} - \beta_{ir} - 1}{(1-\alpha_{ir})a^*} < 0 \tag{31}$$

$$\frac{\partial^2 \Pi R_r(k_{ir})}{\partial k_{ir}^2} = -\frac{Th_{ir}D_{ir}}{k_{ir}^3} < 0 \tag{32}$$

Since Equations (31) and (32) according to assumptions is negative, then objective function is concave, and proof is completed. □

(iii) If f is concave function then f is convex function [5]. Thereby, we prove Hessian of f is a positive semi-definite.

$$H(-\Pi_M(\psi_i, T)) = \begin{bmatrix} \frac{-\partial^2 \Pi_M(\psi_i,T)}{\partial \psi_i^2} & \frac{-\partial^2 \Pi_M(\psi_i,T)}{\partial \psi_i \partial T} \\ \frac{-\partial^2 \Pi_M(\psi_i,T)}{\partial \psi_i \partial T} & \frac{-\partial^2 \Pi_M(\psi_i,T)}{\partial T^2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{2\sum_{i=1}^n As_i}{T^3} \end{bmatrix} \tag{33}$$

Equation (33) is a positive semi-definite if for every non-zero vector (ψ_i, T) Equation (34) is satisfied.

$$(\psi_i, T).\left(H(-\Pi_M(\psi_i, T))\right).(\psi_i, T)^T \geq 0 \tag{34}$$

Here $(\psi_i, T)^T$ denotes the transpose of (ψ_i, T). Therefore,

$$\begin{bmatrix} 0 & \frac{2\sum_{i=1}^n As_i}{T^2} \end{bmatrix} . \begin{bmatrix} \psi_i \\ T \end{bmatrix} = \frac{2\sum_{i=1}^n As_i}{T} \geq 0 \tag{35}$$

According to assumption Equation (24) is satisfied and proof is completed. □

(iv) Substituting Equation (11) into (10) gives,

$$\begin{aligned}\Pi_{S_s}(F_{js}) = &\sum_{j=1}^{J}\sum_{s=1}^{S} F_{js}\left(Q_j - \eta_{js}F_{js} + \sum_{s=1/s}^{S}\theta_{j\underline{s}}F_{j\underline{s}}\right)\\ &-\sum_{j=1}^{J}\sum_{s=1}^{S} Cs_{js}\left(Q_j - \eta_{js}F_{js} + \sum_{s=1/s}^{S}\theta_{j\underline{s}}F_{j\underline{s}}\right)\end{aligned} \tag{36}$$

The second order condition for $\Pi_{S_s}(F_{js})$ in Equation (34),

$$\frac{\partial^2\Pi_{S_s}(F_{js})}{\partial F_{js}^2} = -2\eta_{js} \leq 0 \tag{37}$$

Obviously, according to Equation (37) is non-positive, and the proof is completed. □

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