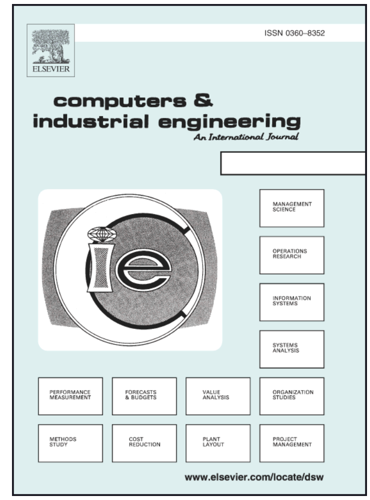
Accepted Manuscript



Coordination via cooperative advertising and pricing in a manufacturer-retailer supply chain

Jafar Chaeb, Morteza Rasti-Barzoki

PII: S0360-8352(16)30232-7
DOI: http://dx.doi.org/10.1016/j.cie.2016.07.007
Reference: CAIE 4404

To appear in: *Computers & Industrial Engineering*

Received Date: 19 September 2015
Revised Date: 16 May 2016
Accepted Date: 7 July 2016

Please cite this article as: Chaeb, J., Rasti-Barzoki, M., Coordination via cooperative advertising and pricing in a manufacturer-retailer supply chain, *Computers & Industrial Engineering* (2016), doi: http://dx.doi.org/10.1016/j.cie.2016.07.007

Coordination via cooperative advertising and pricing in a manufacturer-retailer supply chain

Jafar Chaeb[*1], *Morteza Rasti-Barzoki*[2]

[1] Grad. Student, Department of Industrial Engineering, University of Tehran, Tehran, Iran

[2,] Assistant professor, Department of Industrial & Systems Engineering, Isfahan University of Technology, Isfahan, 84156-8311, Iran

[*] Corresponding author.Tel.: +98 21 22923140; fax: +98 21 88021067.
E-mail address: jafar.chaeb@ut.ac.ir

Coordination via cooperative advertising and pricing in a manufacturer-retailer supply chain

A B S T R A C T

The manufacturer participating in a cooperative advertising scheme reimburses a percentage of local advertising expenditures to encourage the retailer into more promotional initiatives. The present study aims to investigate the supply chain coordination through cooperative advertising and pricing by proposing a relatively general consumer demand function. Based on the underlying balance of power among supply chain members, four possible game structures are discussed including the Nash, Stackelberg retailer, Stackelberg manufacturer and cooperation games. Moreover, numerical simulations are provided to exemplify implicit optimal solutions of the Stackelberg retailer-manufacturer games while they will also be used for comparison of the four games. The unprecedented results obtained from this study may be summarized as follows: 1) the cooperation game is strongly found to be infeasible depending on the certain channel's parameters; 2) contrary to previous findings, the manufacturer's margin is found to be always lower than the retailer's in the Stackelberg retailer game; 3) in the Stackelberg manufacturer game, the manufacturer prefers to advertise nationally rather than to support local promotional activities when retailer advertising becomes inefficient; 4) we find that the manufacturer`s price is entirely stable compared to classical linear model and increases as effectiveness ratio of national to local advertising increases.

Keywords: Supply chain Management, Cooperative advertising, Pricing, Game theory.

1. Introduction

Supply chain management has received significant attention in business and academics from varied disciplines: supply chain contracts, logistics, purchasing, advertising, inventory and pricing (Arshinder et al., 2008; Chen, 2015; Heydari, 2014; Xiao et al., 2014). It is well documented in the literature that the supply chain coordination through either cooperative advertising (co-op) or pricing leads to better performance in distribution channels (Berger, 1972; Choi, 1991; Dant & Berger, 1996; Jeuland & Shugan, 1983; Steffen Jørgensen & Zaccour, 2003a; Somers et al., 1990; Yue et al., 2006). Cooperative advertising and pricing strategies play significant roles in marketing programs of channel members in a supply chain. The total expenditures of cooperative advertising range from $900 million in 1970 to more than $50 billion in 2012, indicating the growing significance of this marketing program (Aust & Buscher, 2013; Nagler, 2006). In addition, the US National Federation of Independent Business estimates an annual amount of nearly $50 billion offered by manufacturers to retailers as cooperative advertising reimbursement (Kraft & Kamieniecki, 2007). When the manufacturer collaborates with the retailer by reimbursing a percentage of the local advertising cost (known as manufacturer`s participation rate), the retailer will be strongly motivated to increase his contribution to local advertising efforts. Meanwhile, the absence of a coordinated decision system leads to inefficiencies in distribution channels that result from what has come to be known in the literature as the 'double moral hazard' or 'double marginalization' (Spengler, 1950; Tirole, 1989; Zhang & Chen, 2013). The present paper addresses coordination through simultaneous cooperative advertising and pricing in a manufacturer-retailer supply chain.

Cooperative advertising has long been an attractive research topic (Aust & Buscher, 2012, 2014b; Berger, 1972; Dant & Berger, 1996; Eliashberg & Steinberg, 1987; Huang & Li, 2001; Somers et al., 1990; Xie & Wei, 2009) that has received considerable attention by industrialists, especially those in the automobile industry (Green, 2000; Karray & Zaccour, 2007). Berger (1972) was the first to carry out a mathematical study of cooperative advertising in a manufacturer-retailer channel. Dant and Berger (1996) extended Berger's model using the game theory to obtain optimal solutions of channel members in a franchising system. The game theory has become a popular approach to investigating the role of cooperative advertising models in manufacturer-retailer supply chains. The studies in this area are divided in the respective literature into two main categories: static and dynamic game theoretic models. The static models study the co-op advertising in a single period; examples include Berger (1972); Dant and Berger (1996); Bergen and John (1997); Huang and Li (2001); Huang et al. (2002); Karray and Zaccour (2006); and Xie and Neyret (2009). Huang and Li (2001) explored the efficiency of co-op advertising regarding transactions in manufacturer-retailer channels. They discussed three co-op advertising models using game theory and applied the Nash bargaining model to determine the sharing rules of advertising expenses. Huang et al. (2002) and Li et al. (2002) adopted similar approaches by considering the impact of brand name investments, local advertising, and sharing policies of advertising expenses in order to study cooperative advertising in a manufacturer-retailer supply chain. They utilized Eliashberg (1986) cooperative bargaining model to show how the channel members jointly divide the extra profits. Yue et al. (2006) extended over the models proposed in Huang and Li (2001), Huang et al. (2002), and Li et al. (2002) by introducing the price discount factor along with the advertising impact when only the manufacturer provides a price deduction directly to customers. The negativity problem of sales volume in the four recent papers mentioned above was corrected in Ahmadi-Javid and Hoseinpour (2011, 2012) who developed a modified version of the model by incorporating two constraints previously suggested in Yue et al. (2006). Using this new version, they found that no variations in the marginal profits of either the manufacturer or the retailer would affect advertising expenditures. More recently, Yue et al. (2013) extended their previous work Yue et al. (2006) to a situation in which both the manufacturer and the retailer offer a price discount to the customer in order to obtain better insights into the underlying relationships of pricing and cooperative advertising.

The second class of game theoretic models, i.e., dynamic models, considers the long–term perspective affecting consumer's goodwill through national and local advertising efforts. Studies in this class include Chintagunta and Jain (1992), Steffen Jørgensen et al. (2000), Steffen Jørgensen et al. (2001), Steffen Jørgensen and Zaccour (2003b), Karray and Zaccour (2005), and He et al. (2009), He et al. (2011). Although Chintagunta and Jain (1992) studied a dynamic model taking into account only the dynamic effects of channel members' advertising efforts, Steffen Jørgensen et al. (2000) and Steffen Jørgensen et al. (2001) extended their model to include a cooperative advertising environment where both short and long-term impacts of advertising efforts boost up sales and consumer goodwill. Karray and Zaccour (2005) extended the models developed by Steffen Jørgensen et al. (2000) and S. Jørgensen et al. (2001) to show how the manufacturer could employ the cooperative advertising strategy for reducing the negative effect of the retailer's private label when he sells two products: the manufacturer's and a private label at a lower price. He et al. (2011) and Wang et al. (2011) investigated the cooperative advertising problem with one monopolistic manufacturer and competing duopolistic retailers using the dynamic and static game theoretic models, respectively. Their analysis showed that the competitive behaviors affected the profits of all the channel members, which motivated them to move to a different game structure. Further, Alaei et al. (2014), Giri and Sharma (2014) and Karray and Amin (2015) examined the cooperative advertising scheme in a channel with single manufacturer and two retailers under different game structures. In addition, the impact of price elasticity on pricing and cooperative advertising decisions has been addressed by Zhao et al. (2015) considering Stackelberg manufacturer game.

Several attempts have been made to study the different factors that influence sales volume; these include national advertising, local promotions, participation rate, retail price, and price deduction in the supply chain coordination settings. Manufacturer`s national brand name investment and retailer's local advertising are two common types of advertising strategies; the former aims to reinforce the brand image and influence the potential consumers, while the latter is intended to induce short-term sales with the aid of local promotional initiatives. A number of recent studies have been undertaken aimed at developing a model that comprises most of the above mentioned factors in order to examine the manufacturer-retailer relationships in a supply chain (Aust & Buscher, 2012; Huang et al., 2002; SeyedEsfahani et al., 2011; Szmerekovsky & Zhang, 2009; Xie & Neyret, 2009; Yue et al., 2006). For instance, Yue et al. (2006) and Szmerekovsky and Zhang (2009) extended Huang et al. (2002) by examining different ways of integrating national and local advertising with price sensitivity impacts and explored the relationships between these factors and the expected market demand. Xie and Wei (2009) developed two models including conflict and cooperation situations where the consumer demand function is determined by both the retail price and the cooperative advertising efforts. A similar approach was adopted by Xie and Neyret (2009). They reviewed the four game theoretic models to identify optimal cooperative advertising and pricing policies. SeyedEsfahani et al. (2011) built upon Xie and Wei (2009) by incorporating a price elasticity (ν) impact that yields a convex ($\nu < 1$), linear ($\nu = 1$), or concave ($\nu > 1$) price demand curve in the four game scenarios. The restrictive assumption of equal margins in the Nash and Stackelberg retailer games was relaxed in Aust and Buscher (2012) by incorporating the models proposed by SeyedEsfahani et al. (2011) and Xie and Wei (2009).

In this paper, we develop four models in which the consumer demand function is simultaneously affected by retail price and cooperative advertising efforts. Our objective is to investigate the underlying interactions among the channel members in a manufacturer-retailer distribution channel. The current study is not only closely connected to the three papers cited above, namely Xie and Neyret (2009), SeyedEsfahani et al. (2011), and Aust and Buscher (2012), but also extends beyond by generating a number of insights. The work of Xie and Neyret (2009) is extended by considering a general price demand function and relaxing the assumption of equal margins in order to understand pricing impacts on channel members' profits. The general price demand function employed is the one proposed in SeyedEsfahani et al. (2011) which may lead to one of the convex ($\nu < 1$), linear ($\nu = 1$), or concave ($\nu > 1$) curves. As Piana (2004) points out, a convex demand curve arises from a polarized distribution of reserve prices (maximum acceptable price) with most consumers having low reserve prices, few are rich, and only slightly more are in the middle . A uniform distribution of reserve prices brings about a linear demand curve and, a distribution of reserve price with a wide number of consumers having a similar middle reserve price, only few rich and few poor, gives rise to concave demand curve. Relaxing the equal margins assumption has also been proposed by Aust and Buscher (2012) who adopted the model proposed in SeyedEsfahani et al. (2011). The last two papers just mentioned are similar to the work by Xie and Wei (2009) in that they employ the same model to address advertising effects on the consumer demand function. Finally, all these papers employed models which have been rarely ever reported in the literature (See Table 1). In contrast, we address the advertising-sales response function by utilizing the model proposed by Huang and Li (2001) which is very popular in the literature (Huang et al., 2002; Li et al., 2002; Szmerekovsky & Zhang, 2009; Xie & Ai, 2006; Xie & Neyret, 2009; Yue et al., 2013; Yue et al., 2006). This advertising-sales response function formulates the relation between national and local advertising in a multiplicative form compared to square root model shown in Table 1 that works in an additive way. Note that it is critical to determine how the market demand depends on national and local advertising (Zhao et al., 2015). It is also superior in terms of speed control of converging to the saturation level compared to the square root advertising demand function (Aust & Buscher, 2014a).

Therefore, the two assumptions of linear price demand function and restrictive equal margins are relaxed in an attempt to revisit the supply chain coordination in the manufacturer-retailer distribution channel in the light of findings reported in the more recent literature. Choi (1991) asserted that different demand functions produce different results, some of which crucially depend on the form of the consumer demand function. Our findings differ significantly from those reported elsewhere.

Table 1
The most related cooperative advertising models and the proposed model

	Equality of margins assumption	Price demand	Advertising demand	Game structures
Huang and Li (2001)	----	----	$\alpha - \beta a^{-\gamma} q^{-\delta}$	N, SM and Co
Xie and Wei (2009)	----	$1 - \beta p$	$k_r\sqrt{a} + k_m\sqrt{q}$	SM and Co
Xie and Neyret (2009)	*Assumed*	$\alpha - \beta p$	$A - Ba^{-\gamma} q^{-\delta}$	N, SR, SM and Co
SeyedEsfahani et al. (2011)	*Assumed*	$(\alpha - \beta p)^{\frac{1}{\nu}}$	$k_r\sqrt{a} + k_m\sqrt{q}$	N, SR, SM and Co
Aust and Buscher (2012)	*Relaxed*	$(\alpha - \beta p)^{\frac{1}{\nu}}$	$k_r\sqrt{a} + k_m\sqrt{q}$	N, SR, SM and Co
Proposed Model	*Relaxed*	$(\alpha - \beta p)^{\frac{1}{\nu}}$	$A - Ba^{-\gamma} q^{-\delta}$	N, SR, SM and Co

The rest of the paper is organized as follows. In the next Section, we introduce the model and describe the problem. Section 3 presents the four different game structures in a manufacturer-retailer relationship. Numerical simulations are presented in Section 4, which are meant to illustrate the application of the proposed model and to compare the four games considered. Section 5 provides explanations on the feasibility conditions of the cooperation game. Discussions and comparisons are presented in Section 6. Finally, Section 7 concludes the paper with remarks and implications.

2. Model development and notations

Consider a two-member supply chain where one manufacturer wholesales a product to one retailer who sells the manufacturer's product to consumers. Table 2 provides the notations used for the decision variables and parameters of the problem.

Table 2
Notations

Decision variables		Parameters	
w	Wholesale price	α	Price demand potential
q	National advertising	β	Price sensitivity
t	Manufacturer`s participation rate	ν	Shape parameter
p	Retail price	γ	Effectiveness of local advertising
m	Retailer margin	δ	Effectiveness of national advertising
a	Local advertising	A	Sales saturate asymptote
Π_M	Manufacturer`s profit	B	Advertising sensitivity
Π_R	Retailer`s profit	c	Manufacturer`s unit production cost
Π_{M+R}	System`s profit	d	Retailer`s unit handling cost

The consumer demand function depends on both marketing efforts and retail price in a multiplicative form; this is a well-known function in the literature (S. Jørgensen & Zaccour, 1999; Xie & Wei, 2009; Yue et al., 2006) repeated below:

$$V(p,a,q) = g(p)S(a,q) \tag{1}$$

where, the advertising-sales response function, as also proposed by Huang and Li (2001), is of the form: $S(a,q) = A - Ba^{-\gamma}q^{-\delta}$, which represents an increasing and concave function to reproduce the diminishing returns. We also employ $g(p) = (\alpha - \beta p)^{\frac{1}{\nu}}$ as the price demand function similar to the approach adopted by SeyedEsfahani et al. (2011), depending on whether the shape parameter ν yields a convex, linear, or concave curve. This gives us:

$$V(p,a,q) = (\alpha - \beta p)^{\frac{1}{\nu}} (A - Ba^{-\gamma}q^{-\delta}) \tag{2}$$

In order to relax the equal margins assumption, we also use Aust and Buscher (2012) method in applying a new decision variable denoted by m as the retailer margin; that is:

$$m = p - w \tag{3}$$

Finally, the functions of the manufacturer's profit, the retailer's profit, and the system's profit can be written as follows:

$$\Pi_M = (w-c)(\alpha - \beta(m+w))^{\frac{1}{\nu}}\left(A - \frac{B}{a^{\gamma} q^{\delta}}\right) - ta - q \tag{4}$$

$$\Pi_R = (m-d)(\alpha - \beta(m+w))^{\frac{1}{\nu}}\left(A - \frac{B}{a^{\gamma} q^{\delta}}\right) - (1-t)a \tag{5}$$

$$\Pi_{M+R} = \big(p - (c+d)\big)(\alpha - \beta(m+w))^{\frac{1}{\nu}}\left(A - \frac{B}{a^{\gamma} q^{\delta}}\right) - a - q \tag{6}$$

In order to avoid negativity of demand function, the following conditions should be met:

$$V(p,a,q) > 0 \;\Rightarrow p < \frac{\alpha}{\beta} \text{ or } w < {}^{\alpha}/_{\beta} - m, A - \frac{B}{a^{\gamma} q^{\delta}} \geq 0, \tag{7}$$

$w > c$, $m > d$ and $p > c + d$.

Yue et al. (2006) claim that the function $A - \frac{B}{a^{\gamma} q^{\delta}}$ is valid only when the national and local advertising expenditures are no less than some positive values; i.e., $a \geq a_0$ and $q \geq q_0$. These constraints are incorporated by Ahmadi-Javid and Hoseinpour (2012) into the model of Huang and Li (2001) in an attempt to obtain different results. However, Yue et al. (2006) and Yue et al. (2013) did not employ the above-mentioned constraints since they intended to ease the study of the co-op advertising model. Neither will we consider this problem so as to simplify the analysis of manufacturer-retailer relationships in our proposed model. A recent survey by Steffen Jørgensen and Zaccour (2014) on game-theoretic models in corresponding literature has shown that these constraints are seen to be ad hoc and more research has to be conducted to solve the problem of negative demand. Following Xie and Neyret (2009), we will employ an appropriate change of variables to handle the problem in an equivalent but more convenient way shown in Table 3.

Table 3
Change of variables

$\alpha' = \alpha - \beta(c+d) > 0$	$a' = \frac{a}{B'^{\frac{1}{\delta+\gamma+1}}}$
$w' = \frac{\beta}{\alpha'}(w-c) > 0$	$q' = \frac{q}{B'^{\frac{1}{\delta+\gamma+1}}}$
$m' = \frac{\beta}{\alpha'}(m-d) > 0$	$\Pi_M' = \frac{\Pi_M}{B'^{\frac{1}{\delta+\gamma+1}}}$
$p' = \frac{\beta}{\alpha'}\big(p-(c+d)\big) > 0$	$\Pi_R' = \frac{\Pi_R}{B'^{\frac{1}{\delta+\gamma+1}}}$
$A' = \frac{\alpha^{\frac{1}{\nu}+1}}{\beta} A$	$\Pi_{M+R}' = \frac{\Pi_{M+R}}{B'^{\frac{1}{\delta+\gamma+1}}}$
$B' = \frac{\alpha^{\frac{1}{\nu}+1}}{\beta} B$	

According to Table 3 , we will have:

$p < \frac{\alpha}{\beta} \Longrightarrow \beta p - \beta(c+d) < \alpha - \beta(c+d) \Longrightarrow \frac{\beta p - \beta(c+d)}{\alpha - \beta(c+d)} < 1 \;\; \Longrightarrow p' < 1 \;\Longrightarrow$
$1 - (w' + m') > 0$

Firstly, the change of variables on the left hand side of Table 3 is applied to obtain the following relations:

$$\Pi_M = w'\left(1 - (w' + m')\right)^{\frac{1}{\nu}}\left(A' - \frac{B'}{a^{\gamma}q^{\delta}}\right) - ta - q,$$

$$\Pi_R = m'\left(1 - (w' + m')\right)^{\frac{1}{\nu}}\left(A' - \frac{B'}{a^{\gamma}q^{\delta}}\right) - (1-t)a,$$

$$\Pi_{M+R} = p'\left(1 - (w' + m')\right)^{\frac{1}{\nu}}\left(A' - \frac{B'}{a^{\gamma}q^{\delta}}\right) - a - q.$$

Then, the change of variables on the right hand side of Table 3 is effected to express the manufacturer's, the retailer's and the system's profits as follows:

$$\Pi'_M = w'\left(1 - (w' + m')\right)^{\frac{1}{\nu}}\left(\frac{A'}{B'^{\frac{1}{\delta+\gamma+1}}} - \frac{1}{a'^{\gamma}q'^{\delta}}\right) - ta' - q', \tag{8}$$

$$\Pi'_R = m'\left(1 - (w' + m')\right)^{\frac{1}{\nu}}\left(\frac{A'}{B'^{\frac{1}{\delta+\gamma+1}}} - \frac{1}{a'^{\gamma}q'^{\delta}}\right) - (1-t)a', \tag{9}$$

$$\Pi'_{M+R} = p'\left(1 - (w' + m')\right)^{\frac{1}{\nu}}\left(\frac{A'}{B'^{\frac{1}{\delta+\gamma+1}}} - \frac{1}{a'^{\gamma}q'^{\delta}}\right) - a' - q'. \tag{10}$$

Henceforth, we will eliminate the superscript ($'$) for the sake of simplicity.

3. The four game structures of the channel members

3.1. Nash game

In this section, it is assumed that the two supply chain members make the decisions independently and simultaneously to maximize their own individual profits. The solution of this structure is obtained by using the Nash game where both players have non-cooperatively equal powers in the distribution channel. To determine the Nash equilibrium point, we model the manufacturer's and the retailer's problems as shown below:

Max $\quad \Pi_M = w\left(1 - (w + m)\right)^{\frac{1}{\nu}}\left(\frac{A}{B^{\frac{1}{\delta+\gamma+1}}} - \frac{1}{a^{\gamma}q^{\delta}}\right) - ta - q$

s.t. $\quad w < 1 - m,\; 0 \le t < 1 \; and \; 0 < q$ (11)

Max $\quad \Pi_R = m\left(1 - (w + m)\right)^{\frac{1}{\nu}}\left(\frac{A}{B^{\frac{1}{\delta+\gamma+1}}} - \frac{1}{a^{\gamma}q^{\delta}}\right) - (1-t)a$

s.t. $\quad m < 1 - w,\; and \; 0 < a$ (12)

By equating the first-order partial derivative of the player's profits to zero, with respect to the relevant decision variables, and also by solving all the derived equations simultaneously, one can obtain the following results from the Nash equilibrium. Note that the optimal value of t is zero because of its negative coefficient in the manufacturer`s objective function.

Theorem 1. *When the channel power is non-cooperatively balanced between the manufacturer and the retailer, the optimal solutions of the channel members are characterized by the Nash equilibrium as follows:*

$$w^N = m^N = \frac{\nu}{1+2\nu}, \qquad p^N = \frac{2\nu}{1+2\nu}, \qquad t^N = 0, \tag{13}$$

$$a^N = \left(\frac{\left(\frac{\gamma}{\delta}\right)^{\delta}\gamma\nu}{(1+2\nu)^{\frac{1}{\nu}+1}}\right)^{\frac{1}{\delta+\gamma+1}}, \qquad q^N = \frac{\delta}{\gamma}a^N. \tag{14}$$

Proof. The proofs for this and other theorems in this paper are presented in the Appendix (in the online version) to save space.

3.2. Stackelberg retailer game

When the retailer has a dominant power in the manufacturer–retailer channel, the retailer as the leader imposes his decisions on the manufacturer who is considered to be the follower. This situation is called the 'Stackelberg retailer game' in which the channel members determine the Stackelberg retailer equilibrium in a sequential non-cooperative game. At this point, the retailer maximizes his own profits with regard to the best manufacturer's response to his decisions. Thus, we need to solve the decision problem of the manufacturer and, subsequently, incorporate the manufacturer's response into the retailer's decision problem in order to determine the channel members' optimal solutions. The best manufacturer's response is obtained from Eq. (12) by calculating $\frac{\partial \Pi_M}{\partial w} = 0$ and $\frac{\partial \Pi_M}{\partial q} = 0$ as follows:

$$w = \frac{\nu(1-m)}{1+\nu}, \quad q = \left(\frac{\delta w\left(1-(m+w)\right)^{\frac{1}{\nu}}}{a^{\gamma}}\right)^{\frac{1}{\delta+1}} \text{ and } t = 0. \tag{15}$$

Therefore, the retailer`s decision problem is expressed by:

$$\text{Max} \qquad \Pi_R = m\left(1 - (w+m)\right)^{\frac{1}{\nu}}\left(\frac{A}{B^{\frac{1}{\delta+\gamma+1}}} - \frac{1}{a^{\gamma}q^{\delta}}\right) - (1-t)a$$

$$\text{s.t.} \qquad w = \frac{\nu(1-m)}{1+\nu}, \; q = \left(\frac{\delta w\left(1-(m+w)\right)^{\frac{1}{\nu}}}{a^{\gamma}}\right)^{\frac{1}{\delta+1}} \text{and } t = 0. \tag{16}$$

After substituting the equivalents of w, q and t in the retailer's objective function, the Stackelberg retailer equilibrium will be obtained by solving $\frac{\partial \Pi_R}{\partial m} = 0$ and $\frac{\partial \Pi_R}{\partial a} = 0$ as follows:

Theorem 2. *When the retailer rises to power as the leader in a manufacturer-retailer distribution channel, the optimal solutions of the channel members are characterized by the Stackelberg retailer equilibrium as follows:*

$$w^{S_R} = \frac{\nu(1-m)}{1+\nu}, \quad p^{S_R} = w^{S_R} + m, \quad t^{S_R} = 0, \tag{17}$$

$$a^{S_R} = \left(\frac{\left(\frac{m\gamma}{\delta+1}\right)^{\delta+1} \left(\frac{1-m}{1+\nu}\right)^{\frac{1-\delta\nu}{\nu}}}{(\delta\nu)^{\delta}} \right)^{\frac{1}{\delta+\gamma+1}}, \qquad q^{S_R} = \frac{\delta(\delta+1)\nu}{m\gamma} \left(\frac{1-m}{1+\nu}\right) a^{S_R}, \tag{18}$$

$$\frac{A}{B^{\frac{1}{\delta+\gamma+1}}} \frac{\nu(1-m)-m}{\nu(1-m)} = \left(\frac{1-m}{1+\nu}\right)^{-\frac{\delta(\nu+1)}{\nu(\delta+1)}} \left(\frac{1}{\left(a^{S_R}\right)^{\gamma} (\delta\nu)^{\delta}}\right)^{\frac{1}{\delta+1}} \left(\frac{\nu(\delta+1)(1-m)-m(1-\delta\nu)}{\nu(\delta+1)(1-m)}\right). \tag{19}$$

The retailer margin m cannot be derived in an explicit form based on the other parameters. It depends on the values of A, B, δ, γ and ν. Once these parameters are determined via numerical simulation, the retailer margin m will be obtained from Eq. (19) and so will the other channel's decision variables. Numerical simulations will be presented in Section 4 to illustrate the optimal solutions and to provide useful insights into the interactions among channel members.

3.3. Stackelberg manufacturer game

This game refers to a common marketing scenario whereby the retailer is dominated by the manufacturer in a distribution channel. The balance of channel power is thus shifted to the manufacturer and its solution is called the Stackelberg manufacturer equilibrium. Similar to the previous subsection, the best retailer's response should be initially determined from Eq. (12) by calculating $\frac{\partial \Pi_R}{\partial m} = 0$ and $\frac{\partial \Pi_R}{\partial a} = 0$. That is:

$$m = \frac{\nu(1-w)}{1+\nu}, \qquad a = \left(m\gamma \frac{\left(1-(m+w)\right)^{\frac{1}{\nu}}}{(1-t)q^{\delta}} \right)^{\frac{1}{\gamma+1}}. \tag{20}$$

The manufacturer's decision problem can then be expressed as follows:

$$\text{Max} \qquad \Pi_M = w\left(1-(w+m)\right)^{\frac{1}{\nu}} \left(\frac{A}{B^{\frac{1}{\delta+\gamma+1}}} - \frac{1}{a^{\gamma} q^{\delta}} \right) - ta - q$$

$$\text{s.t.} \qquad m = \frac{\nu(1-w)}{1+\nu}, \qquad a = \left(m\gamma \frac{\left(1-(m+w)\right)^{\frac{1}{\nu}}}{(1-t)q^{\delta}} \right)^{\frac{1}{\gamma+1}}. \tag{21}$$

In order to determine the Stackelberg manufacturer equilibrium, the manufacturer`s objective function should be maximized subject to the retailer`s response as shown above.

Theorem 3. *When the manufacturer rises to power as the leader in a manufacturer-retailer distribution channel, the optimal solutions of the channel members are characterized by the Stackelberg manufacturer equilibrium in the following two cases:*

Case (i): If $w > \frac{1+2\nu+\nu\gamma}{\nu(1+\gamma)} \Longrightarrow t > 0$, the Stackelberg manufacturer equilibrium is:

$$t^{S_M} = \frac{w(1+2\nu+\nu\gamma)-\nu(1+\gamma)}{w(1+\nu+\nu\gamma)-\nu\gamma}, \qquad m^{S_M} = \frac{\nu(1-w)}{1+\nu}, \qquad p^{S_M} = \frac{\nu+w}{1+\nu}, \tag{22}$$

$$a^{S_M} = \left(\left(\frac{\gamma}{\delta}\right)^{\delta} (\nu(\gamma+1))^{\delta} \left(\frac{1-w}{1+\nu}\right)^{\frac{1+\nu}{\nu}} \left(\frac{\nu\gamma}{1-t}\right)\left\{\frac{(1-w)}{w(1-t)(1+\nu)+t\gamma\nu(1-w)}\right\}^{\delta}\right)^{\frac{1}{\delta+\gamma+1}}, \tag{23}$$

$$q^{S_M} = \frac{\delta}{\gamma}\left(\frac{w(1-t)(1+\nu)+t\gamma\nu(1-w)}{\nu(\gamma+1)(1-w)}\right) a^{S_M}, \tag{24}$$

$$\left(\frac{A}{B^{\frac{1}{\delta+\gamma+1}}}\right)\left(\frac{\nu(1-w)-w}{\nu(1-w)}\right) = \left(\frac{\left(\frac{1-t}{\nu\gamma}\right)^{\gamma}\left(\frac{1-w}{1+\nu}\right)^{-\frac{\gamma(1+\nu)}{\nu}}}{q^{\delta}}\right)^{\frac{1}{\gamma+1}} \left\{\frac{\nu(\gamma+1)(1-w)-w(1-\nu\gamma)}{\nu(\gamma+1)(1-w)} - \frac{t\gamma}{(1-t)(\gamma+1)}\right\}. \tag{25}$$

Case (ii): If $w \leq \frac{1+2\nu+\nu\gamma}{\nu(1+\gamma)} \Longrightarrow t = 0$, the Stackelberg manufacturer equilibrium will be:

$$t^{S_M} = 0, m = \frac{\nu(1-w)}{1+\nu}, p^{S_M} = \frac{\nu+w}{1+\nu}, \tag{26}$$

$$a^{S_M} = \left(\left(\frac{\gamma}{\delta}\right)^{\delta} \left(\frac{\nu(\gamma+1)}{w}\right)^{\delta} \left(\frac{1-w}{1+\nu}\right)^{\frac{1+\nu+\nu\delta}{\nu}} (\nu\gamma)\right)^{\frac{1}{\delta+\gamma+1}}, \tag{27}$$

$$q^{S_M} = \left(\frac{\delta}{\gamma}\right)\left(\frac{w(1+\nu)}{\nu(\gamma+1)(1-w)}\right) a^{S_M}, \tag{28}$$

$$\left(\frac{A}{B^{\frac{1}{\delta+\gamma+1}}}\right)\left(\frac{\nu(1-w)-w}{\nu(1-w)}\right) = \left(\frac{\left(\frac{1-w}{1+\nu}\right)^{-\frac{\gamma(1+\nu)}{\nu}}}{(\nu\gamma)^{\gamma} q^{\delta}}\right)^{\frac{1}{\gamma+1}} \left\{\frac{\nu(\gamma+1)(1-w)-w(1-\nu\gamma)}{\nu(\gamma+1)(1-w)}\right\}. \tag{29}$$

Eqs. (25) and (29) are first-order derivatives of the manufacturer`s objective function with respect to w in which the wholesale price, i.e., w, cannot be determined in a closed form. Unlike the preceding subsections where t was always equal to zero, the optimal value for the participation rate in the Stackelberg manufacturer equilibrium is derived from the above two cases. When $w > \frac{1+2\nu+\nu\gamma}{\nu(1+\gamma)}$**,** the manufacturer's participation rate will be calculated from Case (i) using the optimal manufacturer's price, i.e., w, obtained from Eq. (25) (and based on numerical simulations). For $w \leq \frac{1+2\nu+\nu\gamma}{\nu(1+\gamma)}$, the participation rate will be zero and the optimal solution will be the one presented in Case (ii) in which w is the first decision variable whose value should be established as well.

3.4. The cooperation game

In this subsection, the manufacturer-retailer relationship is perceived as a simultaneous cooperative game where both players are to take steps unanimously in order to maximize the joint total profit. The problem may be characterized as follows:

$$\text{Max} \qquad \Pi_{M+R} = p(1-p)^{\frac{1}{\nu}}\left(\frac{A}{B^{\frac{1}{\delta+\gamma+1}}} - \frac{1}{a^{\gamma}q^{\delta}}\right) - a - q$$

$$\text{s.t.} \qquad 0 < p < 1,\ 0 < q \text{ and } 0 < a. \tag{30}$$

In order to solve this optimization problem, one needs to calculate $\frac{\partial \Pi_{M+R}}{\partial p} = 0, \frac{\partial \Pi_{M+R}}{\partial q} = 0$ and $\frac{\partial \Pi_{M+R}}{\partial a} = 0$.

Theorem 4. *When the channel power is balanced cooperatively between the manufacturer and the retailer, the optimal solutions of the channel members are characterized by the cooperation equilibrium below:*

$$p^{co} = \frac{\nu}{1+\nu}, \qquad a^{co} = \left(\left(\frac{\gamma}{\delta}\right)^{\delta} \frac{\nu\gamma}{(1+\nu)^{\frac{1}{\nu}+1}}\right)^{\frac{1}{\delta+\gamma+1}}, q^{co} = \frac{\delta}{\gamma}a^{co}. \tag{31}$$

Clearly, the optimal values of the retail price and the local and national advertising expenditures are p^{co}, a^{co}, and q^{co}, respectively (See Eq. (31)). However, the optimal values of w, t, and m will be established under the feasibility conditions of the cooperation game stating that the players will agree to make the cooperative decisions if their profits are higher compared to those in the non-cooperative scenarios. Put differently, the manufacturer and the retailer will cooperate only if both win higher individual profits in the cooperation game. In the next Section, a series of numerical simulations will be presented that will enable us to examine the feasibility of the cooperation game

4. Numerical simulations

An approach similar to Xie and Neyret (2009) but more precise is adopted here to present the numerical simulations and to illustrate the previous results. In Eqs. (19), (25), and (29), both the retailer margin and the wholesale price depend on the values of the channel's parameters including δ, γ, ν, A' and B'. Estimating these parameters will be the prerequisite to solving Eqs. (19), (25), and (29). Using the numerical assumptions, the implicit solutions of the retailer margin and the wholesale price presented in the Stackelberg retailer and manufacturer equilibria will be investigated in subsections 4.2 and 4.3, respectively.

4.1. The numerical assumptions

In this subsection, all the parameters remaining after effecting the change of variables presented in Section 2 (i.e., δ, γ, ν, A' and B') will be estimated in order to solve Eqs. (19), (25) and (29). First, the system of Eqs. (6)-(8) is recalled which expresses the channel members' profits. Further, the manufacturer's unit production cost and the retailer's unit handling cost are normalized along the following lines:

$$c = 0,\ d = 0.$$

The sales saturate asymptote A and $\frac{B}{a^{\gamma}q^{\delta}}$ reflect the impacts of advertising efforts in which the former represents maximal expansion of volume sales by advertising expenditures. It is optimistically supposed that A can double sales and $\frac{B}{a^{\gamma}q^{\delta}} = 0.5$. This means that the advertising impact ultimately boost up the sales by 50%; in other words:

$$A = 2 \text{ and } \frac{B}{a^{\gamma}q^{\delta}} = 0.5 \implies \left(A - \frac{B}{a^{\gamma}q^{\delta}}\right) = 1.5 \tag{32}$$

Moreover, Barry and Evans (2012) report that the average advertising expenditures stand for nearly 5% of the net company sales (See pg. 480, "Selected U.S. Advertising-to-Sales Ratios by Type of Retailer"). That is:

$$q + a = \frac{5}{100}\, p(\alpha - \beta p)^{\frac{1}{\nu}}\left(A - \frac{B}{a^{\gamma}q^{\delta}}\right) \tag{33}$$

The retail price p in Eq. (33) needs to be substituted by its equivalent from previous findings for each game. For instance, if we are intended to run the simulation for the cooperation game, the retail price p based on cooperation equilibrium will be as follows:

$$p' = \frac{\nu}{1+\nu} \implies \quad p' = \frac{\beta}{\alpha}p \quad \implies \quad p = \frac{\alpha}{\beta}\left(\frac{\nu}{1+\nu}\right) \tag{34}$$

Based on Eqs. (32) and (34), Eq. (33) may be rewritten as follows:

$$q + a = \frac{5}{100}\frac{\alpha}{\beta}\left(\frac{\nu}{1+\nu}\right)\left(\alpha - \beta\left(\frac{\alpha}{\beta}\left(\frac{\nu}{1+\nu}\right)\right)\right)^{\frac{1}{\nu}}(2 - 0.5) \implies q + a = \frac{3}{40}\frac{\alpha^{\frac{1}{\nu}+1}\nu}{\beta(1+\nu)^{\frac{1}{\nu}+1}} \tag{35}$$

Similarly, the Eq.(35) can be achieved for three other games with respect to corresponding retail price equation. Besides, we may apply the relationships between q and a as are presented in Eqs. (14), (18), (24), (28) and (31) from Nash, Stackelberg retailer, Stackelberg manufacturer and cooperation games, respectively, in order to integrate with Eq.(33) and subsequently obtain q and a with respect to channel`s parameters. The relationship between q and a in cooperation equilibrium is

$$q' = \frac{\delta}{\gamma} a' \Longrightarrow \quad q = \frac{\delta}{\gamma}\, a \tag{36}$$

Solving Eqs. (35)-(36) will, hence, lead to the expression of q and a as follows:

$$a = \frac{\gamma}{\delta+\gamma}\frac{3}{40}\frac{\alpha^{\frac{1}{\nu}+1}\nu}{\beta(1+\nu)^{\frac{1}{\nu}+1}}\,, \qquad\qquad q = \frac{\delta}{\delta+\gamma}\frac{3}{40}\frac{\alpha^{\frac{1}{\nu}+1}\nu}{\beta(1+\nu)^{\frac{1}{\nu}+1}} \tag{37}$$

And, this also yields:

$$B = 0.5\, a^{\gamma} q^{\delta} = 0.5\frac{\delta^{\delta}\gamma^{\gamma}}{(\delta+\gamma)^{\delta+\gamma}}\left(\frac{3}{40}\frac{\alpha^{\frac{1}{\nu}+1}\nu}{\beta(1+\nu)^{\frac{1}{\nu}+1}}\right)^{\delta+\gamma} \tag{38}$$

Finally, by integrating Eq. (38), the results of the numerical simulations in cooperation scenario may be expressed as follows:

$$\frac{A'}{B'^{\frac{1}{\delta+\gamma+1}}} = \frac{A\left(\frac{\alpha^{\frac{1}{\nu}+1}}{\beta}\right)}{B\left(\frac{\alpha^{\frac{1}{\nu}+1}}{\beta}\right)^{\frac{1}{\delta+\gamma+1}}} = \frac{2\left(\frac{\alpha^{\frac{1}{\nu}+1}}{\beta}\right)^{\frac{\delta+\gamma}{\delta+\gamma+1}}}{\left(0.5\frac{\delta^{\delta}\gamma^{\gamma}}{(\delta+\gamma)^{\delta+\gamma}}\left(\frac{3}{40}\frac{\nu}{(1+\nu)^{\frac{1}{\nu}+1}}\right)^{\delta+\gamma}\left(\frac{\alpha^{\frac{1}{\nu}+1}}{\beta}\right)^{\delta+\gamma}\right)^{\frac{1}{\delta+\gamma+1}}},$$

$$\frac{A'}{B'^{\frac{1}{\delta+\gamma+1}}} = 2\left(\frac{2}{\delta^{\delta}\gamma^{\gamma}}\left(\frac{40}{3}\frac{(\delta+\gamma)(1+\nu)^{\frac{1}{\nu}+1}}{\nu}\right)^{\delta+\gamma}\right)^{\frac{1}{\delta+\gamma+1}}. \tag{39}$$

Similarly, Eq.(39) can be obtained for three other non-cooperative games based on respective retail price equation and relationship between q and a. In Xie and Neyret (2009) , both retail price and relationship between q and a are roughly estimated based on results of two games and extended for others. However, we employ optimal solution of every equilibrium for its own game in order to capture much more original results.

Thus, we simply need to estimate the remaining channel`s parameters including δ, γ and ν to gain a better understanding of the Stackelberg retailer and manufacturer equilibria. It is worth stating that the assumptions in this section can be the subject of debate regarding the practical advertising and pricing impacts characterized by Eqs. (32)-(33). However, these assumptions may be easily modified and introduced into the model. What went above was intended to show the application of our relatively general model.

4.2. Numerical results for Stackelberg retailer game

In Subsection 3.2, a closed form was not found for the retailer margin. By applying the numerical assumptions, the retailer margin can be illustrated for different values of δ, γ and ν. In addition, since the remaining channel`s parameters are δ, γ and ν and also because the illustrations cannot be shown when all the three parameters vary simultaneously and continuously, we need to fix one, preferably ν, in order to be able to present the desired results in a 3D representation. Nevertheless, this might not impose any limitations on our analysis due to the fact that one can easily depict the findings for different but fixed values of ν. In this case, Fig. 1 presents the retailer margin when δ and γ vary in the interval $[0.1, 3]$ and ν is fixed to 1. Apparently, the retailer margin is entirely stable as shown in Fig. 1. Additionally, Fig. 2 represents the effects of price elasticity on the behavior of retailer margin. It can be easily seen that the retailer margin increases as the price elasticity increases. Further discussion of the topic will be taken up in the Section on Discussion and Comparisons below.

Fig. 1 The retailer margin when $\nu = 1$

Fig. 2 The effects of price elasticity on retailer margin

4.3. Numerical results for the Stackelberg manufacturer game

For the implicit form of wholesale price, the two alternative situations of $t > 0$ or $t = 0$ were previously established. Fig. 3 shows the wholesale price derived from these two situations presented in Subsection 3.3 with the one alternative that yields maximum manufacturer's profit for $\nu = 1$. While for $\gamma \leq 1.5$, the wholesale price is consistent with the equivalent one in the classical model (Xie & Neyret, 2009), for $\gamma > 1.5$ the results depict a substantial difference indicating that w will decrease as the effectiveness of the retailer's local advertising increases. More precisely, w increases as the ratio of δ to γ increases. This result is completely consistent with Part (i) of Proposition 1 in Xie and Wei (2009).

Similar to the retailer margin, it can be seen again that the behavior of the manufacturer's price curve is entirely stable compared to the one reported in Xie and Neyret (2009). However, the manufacturer`s price is not quite stable when price elasticity increases as shown in Fig. 4. Once again, it can be seen that the manufacturer's price increases as the market demand becomes more sensitive to price changes. In other words, the more elastic the product is, the higher the manufacturer's price will be. In addition, the optimal solution of participation rate is found to be zero when $\nu = 1$ in contrast to equivalent classical model which was obtained positive for $\gamma < 1$.

Fig. 3 Manufacturer's price when $\boldsymbol{\nu = 1}$

Fig. 4 Manufacturer's price when $\boldsymbol{\nu = 5}$

5. Feasibility of cooperation game

In the previous Section, the retailer margin and the wholesale price were illustrated in the Stackelberg retailer and manufacturer games, respectively. Once these decision variables are determined using the numerical simulations, the optimal solutions of the Stackelberg games and channel members' profits can be easily obtained. In continuation, we will endeavor to see whether or not feasible solutions exist in the cooperation game. As already mentioned, the manufacturer and the retailer shift to cooperation if they both achieve higher individual profits. Thus, the following conditions must be satisfied:

$$\Delta\Pi_M = \Pi_M^{co} - \Pi_M^{max} \geq 0, \tag{40}$$

$$\Delta\Pi_R = \Pi_R^{co} - \Pi_R^{max} \geq 0, \tag{41}$$

$$\Delta\Pi_{M+R} = \Delta\Pi_M + \Delta\Pi_R = \Pi_{M+R}^{co} - (\Pi_M^{max} + \Pi_R^{max}) \geq 0, \tag{42}$$

Π_M^{co}, Π_M^{max} and $\Delta\Pi_M$ represent the manufacturer's profits in the cooperation game, the maximum manufacturer's profit within non-cooperative scenarios, and the extra profit gained by shifting into the cooperation game, respectively. Similar designations for the retailer`s and the system`s profits are $\Pi_R^{co}, \Pi_R^{max}, \Pi_{M+R}^{co}, \Delta\Pi_R$ and $\Delta\Pi_{M+R}$. If both Ineqs. (40)-(41) hold true, the feasibility of the cooperation game is established. Therefore, the maximum profit of the channel members should be investigated within the Stackelberg manufacturer, retailer, and Nash equilibria for different values of ν. Table 4 is a summary of the results of the manufacturer's and retailer's maximum profits when ν varies in the interval $[0.1, 10]$. For $\nu < 0.3$, the maximum profit of the retailer is derived from both the Stackelberg retailer and manufacturer cases, depending on the values of δ, γ. Furthermore, the maximum profit of the manufacturer for $\nu < 0.3$ is found in the Stackelberg manufacturer and Nash cases. In addition, the maximum profits of the manufacturer and the retailer were obtained for $\nu \geq 0.3$ when they are leaders.

Table 4
Maximum channel members' profits among the three non-cooperative games

Price Elasticity \ Players' Profit	Π_M^{max}	Π_R^{max}
$\nu < 0.3$	$\Pi_M^{S_M}$ / Π_M^N	$\Pi_R^{S_R}$ / $\Pi_R^{S_M}$
$0.3 \leq \nu \leq 10$	$\Pi_M^{S_M}$	$\Pi_R^{S_R}$

Fig. 5 is derived from $\Delta\Pi_{M+R} = \Pi_{M+R}^{co} - (\Pi_M^{max} + \Pi_R^{max})$ using numerical simulations to investigate the feasibility of the cooperation game. It reveals that the cooperation is strongly infeasible for the most part, which is significantly different from the results reported previously (Aust & Buscher, 2012; SeyedEsfahani et al., 2011; Xie & Neyret, 2009; Xie & Wei, 2009). Xie and Wei (2009) and SeyedEsfahani et al. (2011) found that the cooperation is always feasible while Aust and Buscher (2012) captured the infeasibility only for large values of the shape parameter ν. However, we found that the cooperation can be feasible merely for very low values of δ, γ and ν, which is also different from the findings of Xie and Neyret (2009) who considered the special case of $\nu = 1$.

Fig. 5 The feasibility of the cooperation game

6. Discussions and comparisons

6.1.The margins and prices

The results from the four game models are recalled in the context of the pricing decisions to provide a detailed discussion and comparison. It should be noted that the illustrations are depicted in 3D forms with regard to δ, γ and ν throughout this Section.

We begin by comparing the retailer margins among the four games as expressed in Ineq.(43). It is found that the retailer margin of the Stackelberg retailer game is always higher than those in the other non-cooperative games and the retailer margin of Nash case is "almost always"[1] higher than that of Stackelberg manufacturer game. That is:

$$m^{S_R} > m^N > m^{S_M} \tag{43}$$

Note that in the comparisons of the retailer margins as well as the wholesale prices and the participation rates, the cooperation case is not included. Moreover, the retailer margin is higher when the retailer is the leader rather than in a conflict situation (the Nash game), contrary to the findings of SeyedEsfahani et al. (2011) and more sensibly when compared to the findings of Aust and Buscher (2012). The former study assumed equal margins for both the manufacturer and the retailer in the Stackelberg retailer game. This caused the Nash game to achieve an irrationally higher retailer margin compared to the situation in the Stackelberg game. In the latter work, all the non-cooperative games tend to be in a position of the highest retailer margin among the games considered.

[1] The situation $m^{S_R} > m^{S_M} > m^N$ might hold true in very few sets of parameters δ *and* γ and merely when $\nu < 0.3$, which are quite negligible. Henceforth, we waive these trivial sets and when this is the case, we state "almost always".

In the comparison of the manufacturer's prices, the wholesale price of the Stackelberg manufacturer equilibrium is "almost always" greater than the Nash equilibrium and that of the Nash equilibrium is always greater than the Stackelberg retailer equilibrium. That is:

$$w^{S_M} > w^N > w^{S_R} \tag{44}$$

However, when the effectiveness of local advertising is high and that of national advertising is low, both the wholesale prices of the Nash and Stackelberg retailer equilibria will exceed that of the Stackelberg manufacturer equilibrium only when $\nu < 0.3$.

Within the retail prices of the four games, the lowest is exclusively obtained in the cooperation case which is consistent with the results reported by Aust and Buscher (2012). However, Xie and Neyret (2009) and SeyedEsfahani et al. (2011) found the lowest retail price concurrently in the cooperation game and in the case when the retailer was the leader. In fact, when the equality assumption of the channel members' margins is unrestrained in the Stackelberg retailer game as Aust and Buscher (2012) did, the retailer imposes a higher retail price on the consumer in order to achieve a high margin. Moreover, the highest retail prices occur in the Stackelberg manufacturer and retailer equilibria and these are "almost always" higher than that of the Nash equilibrium (See Fig. 6).

Fig. 6 Comparison of retail prices

This Subsection examines the assumption of equal margins in the Nash and Stackelberg retailer games as employed by Xie and Neyret (2009) and SeyedEsfahani et al. (2011). According to the Nash equilibrium, both the manufacturer's and the retailer's margins are the same, indicating the intrinsic property of the Nash game that results in equal margins for the channel members. However, our results for the Stackelberg retailer game reveal that the assumption of equal margins is no longer acceptable and that the ratio of wholesale to retail price is always less than $1/2$. Fig. 7 depicts this ratio for $\nu = 0.1$ where the retailer gains a higher margin compared to the manufacturer in the Stackelberg retailer game and this ratio rises as ν increases. This is a distinct result compared to those of Xie and Neyret (2009) and SeyedEsfahani et al. (2011) who assumed equal margins and those reported by Aust and Buscher (2012) who found either a higher, equal, or lower ratio between the manufacturer's and the retailer's margins in this game.

Fig. 7 The ratio of wholesale to retail price in the Stackelberg retailer game when $\boldsymbol{\nu = 0.1}$

6.2. Advertising Expenditures

The literature suggests that the highest advertising expenditures occur in a cooperative environment. Similar results have been found using the numerical simulations for national advertising throughout the four game structures. As shown in Fig. 8, the highest national advertising is seen mainly in the cooperation game except when δ and γ are high and the price elasticity is sufficiently low and also in hollow zone. Our computations imply that national advertising in the Stackelberg retailer game may result in the lowest expenditures, especially for the big ν. This is contrary to the findings of Xie and Neyret (2009) and SeyedEsfahani et al. (2011). In addition, the Nash equilibrium never takes the smallest position among the four games in this context.

Fig. 8 Comparison of national advertising expenditures

Many different cases may emerge from the comparison of local advertising expenditures as summarized in Table 5. Each case may reproduce more than one region in the considered area, which makes it too hard to match the cases with relevant corresponding regions via the illustration. Thus, Table 5 has been prepared to show all the possible cases in which the cooperation game mainly represents the highest local advertising expenditures. In addition, the highest local advertising can occur in the Stackelberg retailer and manufacturer equilibria, each in one case, as shown in Table 5.

Table 5
Comparison of local advertising expenditures

Comparison of all possible cases in local advertising expenditures	
$a^{co} > a^{S_M} > a^N > a^{S_R}$	$a^{co} > a^N > a^{S_R} > a^{S_M}$
$a^{co} > a^N > a^{S_M} > a^{S_R}$	$a^{S_M} > a^{co} > a^N > a^{S_R}$
$a^{co} > a^{S_M} > a^{S_R} > a^N$	$a^{S_M} > a^{co} > a^{S_R} > a^N$
$a^{co} > a^{S_R} > a^{S_M} > a^N$	$a^{S_M} > a^{S_R} > a^{co} > a^N$
$a^{co} > a^{S_R} > a^N > a^{S_M}$	$a^{S_R} > a^{co} > a^N > a^{S_M}$

6.3. Participation rate

The optimal solutions of the participation rate will be studied here in the light of previous findings. The manufacturer's participation rate was found to be zero in the Nash and Stackelberg retailer equilibria. In the Stackelberg manufacturer game, the optimal solution of the participation rate has been plotted in Fig. 9 for different values of ν (i.e., $\nu = 3, \nu = 4$ and $\nu = 5$). Clearly, participation rate boosts up as the values for ν and δ increase and that of γ decreases. Moreover, we have obtained two interesting results concerning the manufacturer`s contribution in local advertising expenditures. First of all, the optimal solution of participation rate is found to be zero when the price elasticity is sufficiently low. Second, and most importantly, the manufacturer pays a more portion of retailer`s local promotional costs as effectiveness of local advertising decreases (and ν increases) in order to compensate for low impact of these advertising efforts. In fact, the manufacturer tends to raise the participation rate when either the market demand is more price sensitive or the local advertising is not very effective in the hope to induce more sales (See Fig. 9 for different values of ν). However, the manufacturer continues to keep his contribution until the effectiveness of local advertising will fall to the level whereby the manufacturer decides to stop investing in retailer`s promotional activities. At this point, he will increase the national advertising expenditures to reap a higher profit. In other words, the manufacturer does not participate in local advertising for $\gamma < 1$ as shown in Fig. 9 when he is the leader. Instead, as depicted in Fig. 10 , the national advertising suddenly increases for $\gamma < 1$ which subsequently yields a higher profit compared to the one that was derived from case (i) of Stackelberg manufacturer equilibrium in which the participation rate was positive. Surprisingly, this result has not predicted by previous studies, however, these findings further support the idea that there exists a threshold for the manufacturer`s participation rate to drop suddenly to zero when the retailer`s inefficient local advertising begins to incur a lower profit.

Fig. 9 Participation rate in the Stackelberg manufacturer game

Fig. 10 National advertising in the Stackelberg manufacturer game

6.4. Profits

For all possible scenarios of the manufacturer-retailer relationships, the profits of the manufacturer, the retailer, and the system are critically significant from the viewpoints of the channel members. Here, we compare the profits accrued from the four game theoretic models for the manufacturer, the retailer, and the system. The manufacturer's profit in all the four games "almost always" follows the relationship below:

$$\Pi_M^{S_M} > \Pi_M^N > \Pi_M^{S_R} \tag{45}$$

According to Table 4, for very exceptional sets of parameters, the manufacturer may initially opt for the Nash game. Generally speaking, the manufacturer achieves a higher profit in the Stackelberg manufacturer game than in the Nash one and he "almost always" gains the lowest profit when the retailer is the leader. Indeed, since the retailer imposes a tiny margin on the manufacturer in the Stackelberg retailer game as characterized by Ineq.(44), the manufacturer gains the lowest profit in this situation. In contrast to Xie and Neyret (2009), SeyedEsfahani et al. (2011), and Aust and Buscher (2012) who found the Stackelberg retailer game as the second or even the first choice for the manufacturer to play with the retailer, the unrestricted Stackelberg retailer equilibrium will be the last choice for the manufacturer. This is another piece of evidence indicating the way our proposed model affects the manufacturer-retailer relationships and that it provides a better understanding of the supply chain coordination.

On the other hand, the retailer's profit is "almost always" higher in the Stackelberg retailer equilibrium. In addition, the retailer "almost always" prefers to be in the conflict situation rather than following the manufacturer. This is expressed by:

$$\Pi_R^{S_R} > \Pi_R^N > \Pi_R^{S_M} \tag{46}$$

One can also claim that the retailer's profit is always higher when he is the leader than what it may be in the Nash equilibrium. Like before, for a very few sets of parameters, the retailer's profit in the Stackelberg manufacturer game may exceed those in either of the other two situations.

Because of reproducing different and complex regions in the three dimensional space, it does not interest us to show the illustrations in this case. Instead, Table 6 summarizes all possible cases in comparison of total profits. It is commonly reported in the literature that the system's profit is highest when the channel members move to the cooperation alternative in any of the four games. Our computations imply that the cooperation game mainly produces the highest system`s profit within four games, however, the Stackelberg games can also result in highest total profit specifically when δ and γ are high and price elasticity is sufficiently low. This outcome is significant for the channel members when they fail to design an efficient cooperative advertising plan in cooperative settings and the players may decide to move to another game structure.

Table 6
Comparison of system`s profits

Comparison of all possible cases in system`s profits	
$\Pi_{M+R}^{Co} > \Pi_{M+R}^{S_M} > \Pi_{M+R}^{N} > \Pi_{M+R}^{S_R}$	$\Pi_{M+R}^{Co} > \Pi_{M+R}^{N} > \Pi_{M+R}^{S_R} > \Pi_{M+R}^{S_M}$
$\Pi_{M+R}^{Co} > \Pi_{M+R}^{N} > \Pi_{M+R}^{S_M} > \Pi_{M+R}^{S_R}$	$\Pi_{M+R}^{S_M} > \Pi_{M+R}^{Co} > \Pi_{M+R}^{N} > \Pi_{M+R}^{S_R}$
$\Pi_{M+R}^{Co} > \Pi_{M+R}^{S_M} > \Pi_{M+R}^{S_R} > \Pi_{M+R}^{N}$	$\Pi_{M+R}^{S_M} > \Pi_{M+R}^{Co} > \Pi_{M+R}^{S_R} > \Pi_{M+R}^{N}$
$\Pi_{M+R}^{Co} > \Pi_{M+R}^{S_R} > \Pi_{M+R}^{S_M} > \Pi_{M+R}^{N}$	$\Pi_{M+R}^{S_M} > \Pi_{M+R}^{S_R} > \Pi_{M+R}^{Co} > \Pi_{M+R}^{N}$
$\Pi_{M+R}^{Co} > \Pi_{M+R}^{S_R} > \Pi_{M+R}^{N} > \Pi_{M+R}^{S_M}$	$\Pi_{M+R}^{S_R} > \Pi_{M+R}^{Co} > \Pi_{M+R}^{N} > \Pi_{M+R}^{S_M}$

7. Concluding remarks and implications

This research investigated the role of supply chain coordination through co-op advertising and pricing in the manufacturer-retailer distribution channel. Four game theoretic models were studied based on the supply chain power balance to describe the different relationships among the channel members. In addition, the optimal solutions of the channel members were analytically obtained in the Nash and cooperation games; however, the Stackelberg retailer and manufacturer equilibria seemed too complex to accept explicit solutions. Hence, a series of numerical simulations were presented not only for illustrating the optimal solutions of the channel members under the scenarios investigated but also for interpreting and comparing the underlying results of the four game models.

We developed a consumer demand function popular in the co-op advertising literature proposed by Huang and Li (2001). The present study relaxed the assumptions of linear price demand function and equal margins in the Stackelberg retailer game suggested by SeyedEsfahani et al. (2011) and Aust and Buscher (2012), respectively. However, the main difference between the proposed model and the two models in the works cited above lies in the fact that we applied these approaches to a different but well-known advertising-sales response function and obtained significantly different results. Moreover, the classic model of Xie and Neyret (2009) was extended into a new form in which the special assumptions of linear price demand function and restrictive identical margins in the Nash and Stackelberg retailer games were relaxed.

This work contributes to existing knowledge of supply chain coordination by providing useful insights into the interplay among the channel members. We found that the wholesale price was entirely stable and the optimal participation rate was zero compared to classical model when the manufacturer rose to power. In addition, the evidence from Stackelberg manufacturer equilibrium suggests that there exists a threshold in which the manufacturer offers no advertising support for inefficient local advertising and decides to advertise nationally in order to gains a higher profit. This finding is consistent with Szmerekovsky and Zhang (2009) results in which employed the same advertising-sales response function with nonlinear demand function.

Another distinct contribution of this paper relies on the relationship among the channel members' margins whereby the equal margins never occurred in Stackelberg retailer game and the manufacturer's margin was found to be lower than that of the retailer's. We also showed that the cooperation scenario was a too weak possible game structure to take place between the manufacturer and the retailer and that it would only be feasible when the price elasticity is sufficiently low. This is a striking implication for the channel members when the cooperative environment is no longer feasible and they have to seek for other game structure. Finally, although we found the cooperation game mainly produces the highest system`s profit, the Stackelberg games also led to a higher one for low price elasticity products which might be interesting implication from the viewpoint of Stackelberg leader.

Appendix

The Appendix associated with this article can be found in the online version.

References

Ahmadi-Javid, A., & Hoseinpour, P. (2011). A Game-Theoretic Analysis for Coordinating Cooperative Advertising in a Supply Chain. Journal of Optimization Theory and Applications, 149(1), 138-150.

Ahmadi-Javid, A., & Hoseinpour, P. (2012). On a cooperative advertising model for a supply chain with one manufacturer and one retailer. European Journal of Operational Research, 219(2), 458-466.

Alaei, S., Alaei, R., & Salimi, P. (2014). A game theoretical study of cooperative advertising in a single-manufacturer-two-retailers supply chain. The International Journal of Advanced Manufacturing Technology, 74(1-4), 101-111.

Arshinder, Kanda, A., & Deshmukh, S. G. (2008). Supply chain coordination: Perspectives, empirical studies and research directions. International Journal of Production Economics, 115(2), 316-335.

Aust, G., & Buscher, U. (2012). Vertical cooperative advertising and pricing decisions in a manufacturer–retailer supply chain: A game-theoretic approach. European Journal of Operational Research, 223(2), 473-482.

Aust, G., & Buscher, U. (2013). Cooperative advertising models in supply chain management: A review. European Journal of Operational Research(0).

Aust, G., & Buscher, U. (2014a). Cooperative advertising models in supply chain management: A review. European Journal of Operational Research, 234(1), 1-14.

Aust, G., & Buscher, U. (2014b). Vertical cooperative advertising in a retailer duopoly. Computers & Industrial Engineering, 72(0), 247-254.

Barry, B., & Evans, J. R. (2012). *Retail Management: A Strategic Approach* (12 edition ed.): Prentice Hall.

Bergen, M., & John, G. (1997). Understanding Cooperative Advertising Participation Rates in Conventional Channels. Journal of Marketing Research, 34(3), 357-369.

Berger, P. D. (1972). Vertical Cooperative Advertising Ventures. Journal of Marketing Research, 9(3), 309-312.

Chen, T.-H. (2015). Effects of the pricing and cooperative advertising policies in a two-echelon dual-channel supply chain. Computers & Industrial Engineering, 87, 250-259.

Chintagunta, P. K., & Jain, D. (1992). A Dynamic Model of Channel Member Strategies for Marketing Expenditures. Marketing Science, 11(2), 168-188.

Choi, S. C. (1991). Price Competition in a Channel Structure with a Common Retailer. Marketing Science, 10(4), 271-296.

Dant, R. P., & Berger, P. D. (1996). Modelling Cooperative Advertising Decisions in Franchising. The Journal of the Operational Research Society, 47(9), 1120-1136.

Eliashberg, J. (1986). Arbitrating a Dispute: A Decision Analytic Approach. Management Science, 32(8), 963-974.

Eliashberg, J., & Steinberg, R. (1987). Marketing-Production Decisions in an Industrial Channel of Distribution. Management Science, 33(8), 981-1000.

Giri, B., & Sharma, S. (2014). Manufacturer's pricing strategy in a two-level supply chain with competing retailers and advertising cost dependent demand. Economic Modelling, 38, 102-111.

Green, J. (2000). Still pulling the strings, but locally too. Brandweek, 41, 34-42.

He, X., Krishnamoorthy, A., Prasad, A., & Sethi, S. P. (2011). Retail competition and cooperative advertising. Operations Research Letters, 39(1), 11-16.

He, X., Prasad, A., & Sethi, S. P. (2009). Cooperative Advertising and Pricing in a Dynamic Stochastic Supply Chain: Feedback Stackelberg Strategies. Production and Operations Management, 18(1), 78-94.

Heydari, J. (2014). Supply chain coordination using time-based temporary price discounts. Computers & Industrial Engineering, 75, 96-101.

Huang, Z., & Li, S. X. (2001). Co-op advertising models in manufacturer–retailer supply chains: A game theory approach. European Journal of Operational Research, 135(3), 527-544.

Huang, Z., Li, S. X., & Mahajan, V. (2002). An Analysis of Manufacturer-Retailer Supply Chain Coordination in Cooperative Advertising*. Decision Sciences, 33(3), 469-494.

Jeuland, A. P., & Shugan, S. M. (1983). Managing Channel Profits. Marketing Science, 2(3), 239-272.

Jørgensen, S., Sigué, S. P., & Zaccour, G. (2000). Dynamic cooperative advertising in a channel. Journal of Retailing, 76(1), 71-92.

Jørgensen, S., Sigué, S. P., & Zaccour, G. (2001). Stackelberg leadership in a marketing channel. International Game Theory Review, 3(1), 13-26.

Jørgensen, S., Taboubi, S., & Zaccour, G. (2001). Cooperative Advertising in a Marketing Channel. Journal of Optimization Theory and Applications, 110(1), 145-158.

Jørgensen, S., & Zaccour, G. (1999). Equilibrium Pricing and Advertising Strategies in a Marketing Channel. Journal of Optimization Theory and Applications, 102(1), 111-125.

Jørgensen, S., & Zaccour, G. (2003a). Channel coordination over time: incentive equilibria and credibility. Journal of Economic Dynamics and Control, 27(5), 801-822.

Jørgensen, S., & Zaccour, G. (2003b). A differential game of retailer promotions. Automatica, 39(7), 1145-1155.

Jørgensen, S., & Zaccour, G. (2014). A survey of game-theoretic models of cooperative advertising. European Journal of Operational Research, 237(1), 1-14.

Karray, S., & Amin, S. H. (2015). Cooperative advertising in a supply chain with retail competition. International Journal of Production Research, 53(1), 88-105.

Karray, S., & Zaccour, G. (2005). A Differential Game of Advertising for National and Store Brands. In A. Haurie & G. Zaccour (Eds.), *Dynamic Games: Theory and Applications* (pp. 213-229): Springer US.

Karray, S., & Zaccour, G. (2006). Could co-op advertising be a manufacturer's counterstrategy to store brands? Journal of Business Research, 59(9), 1008-1015.

Karray, S., & Zaccour, G. (2007). Effectiveness of coop advertising programs in competitive distribution channels. International Game Theory Review, 09(02).

Kraft, M. E., & Kamieniecki, S. (2007). Business and environmental policy: Corporate interests in the American political system. Cambridge: The MIT Press.

Li, S. X., Huang, Z., Zhu, J., & Chau, P. Y. K. (2002). Cooperative advertising, game theory and manufacturer–retailer supply chains. Omega, 30(5), 347-357.

Nagler, M. (2006). An exploratory analysis of the determinants of cooperative advertising participation rates. Marketing Letters, 17(2), 91-102.

Piana, V. (2004). ‘Consumer Decision Rules for Agent-Based Models. Economics Web Institute.

SeyedEsfahani, M. M., Biazaran, M., & Gharakhani, M. (2011). A game theoretic approach to coordinate pricing and vertical co-op advertising in manufacturer–retailer supply chains. European Journal of Operational Research, 211(2), 263-273.

Somers, T. M., Gupta, Y., & Harriot, S. R. (1990). Analysis of cooperative advertising expenditures: a transfer-function modeling approach. Journal of Advertising Research, 30(1), 35-45.

Spengler, J. (1950). Vertical integration and anti-trust policy. Journal of Political Economy, 58, 347-552.

Szmerekovsky, J. G., & Zhang, J. (2009). Pricing and two-tier advertising with one manufacturer and one retailer. European Journal of Operational Research, 192(3), 904-917.

Tirole, J. (1989). The Theory of Industrial Organization. MIT Press, Cambridge MA.

Wang, S.-D., Zhou, Y.-W., Min, J., & Zhong, Y.-G. (2011). Coordination of cooperative advertising models in a one-manufacturer two-retailer supply chain system. Computers & Industrial Engineering, 61(4), 1053-1071.

Xiao, T., Shi, J., & Chen, G. (2014). Price and leadtime competition, and coordination for make-to-order supply chains. Computers & Industrial Engineering, 68, 23-34.

Xie, J., & Ai, S. (2006). A note on “Cooperative advertising, game theory and manufacturer–retailer supply chains”. Omega, 34(5), 501-504.

Xie, J., & Neyret, A. (2009). Co-op advertising and pricing models in manufacturer–retailer supply chains. Computers & Industrial Engineering, 56(4), 1375-1385.

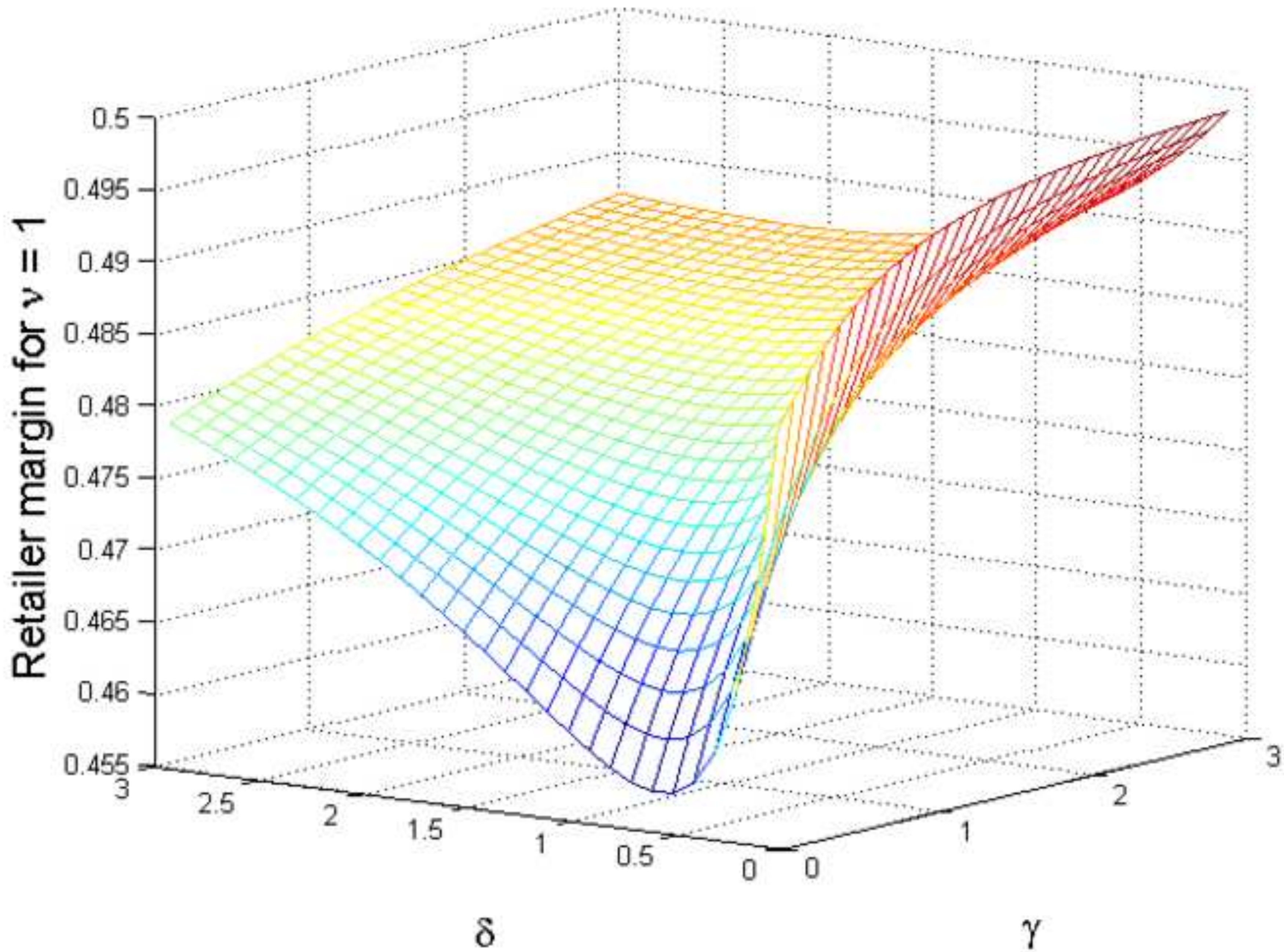
Xie, J., & Wei, J. C. (2009). Coordinating advertising and pricing in a manufacturer–retailer channel. European Journal of Operational Research, 197(2), 785-791.

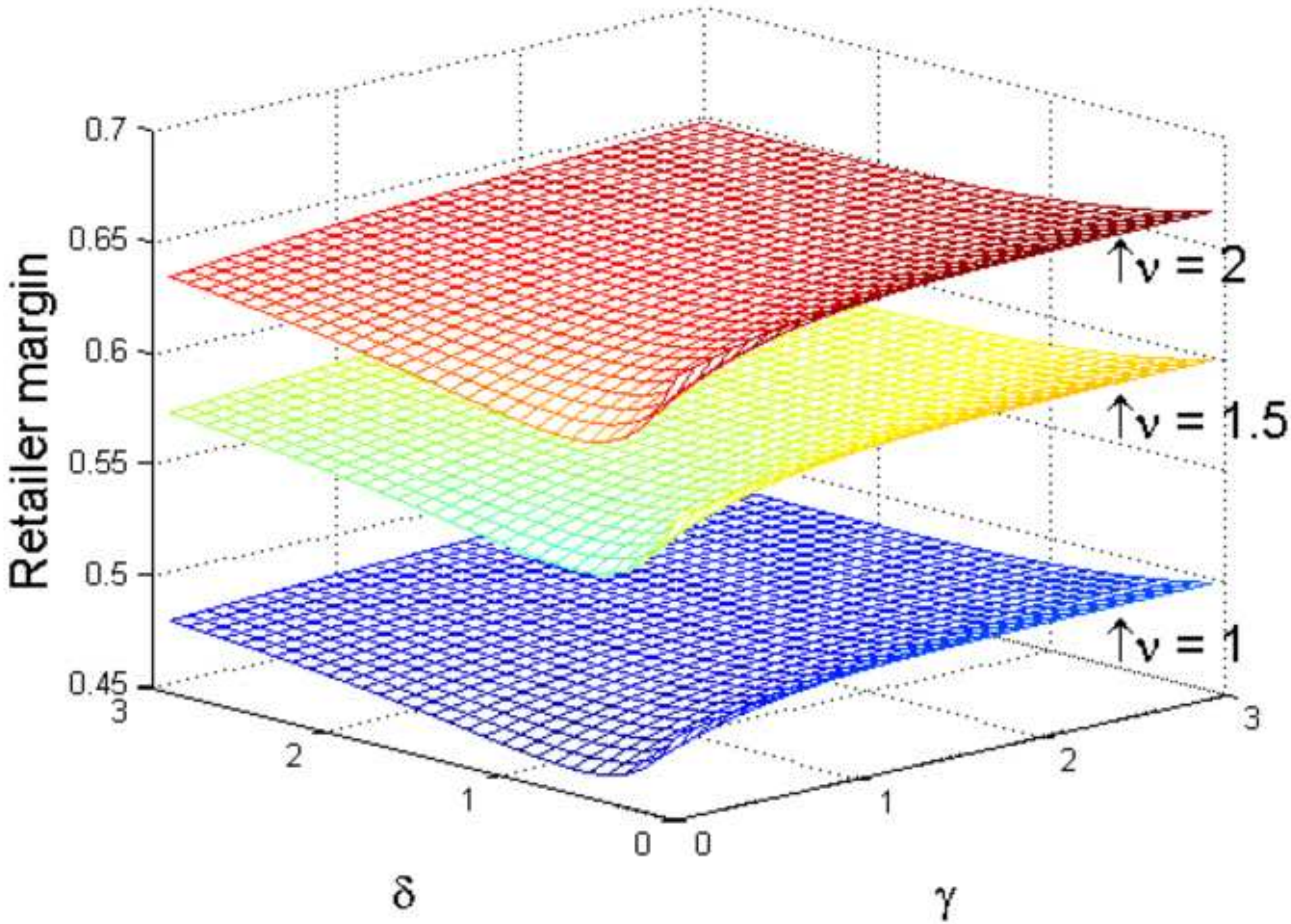
Yue, J., Austin, J., Huang, Z., & Chen, B. (2013). Pricing and advertisement in a manufacturer–retailer supply chain. European Journal of Operational Research, 231(2), 492-502.

Yue, J., Austin, J., Wang, M.-C., & Huang, Z. (2006). Coordination of cooperative advertising in a two-level supply chain when manufacturer offers discount. European Journal of Operational Research, 168(1), 65-85.

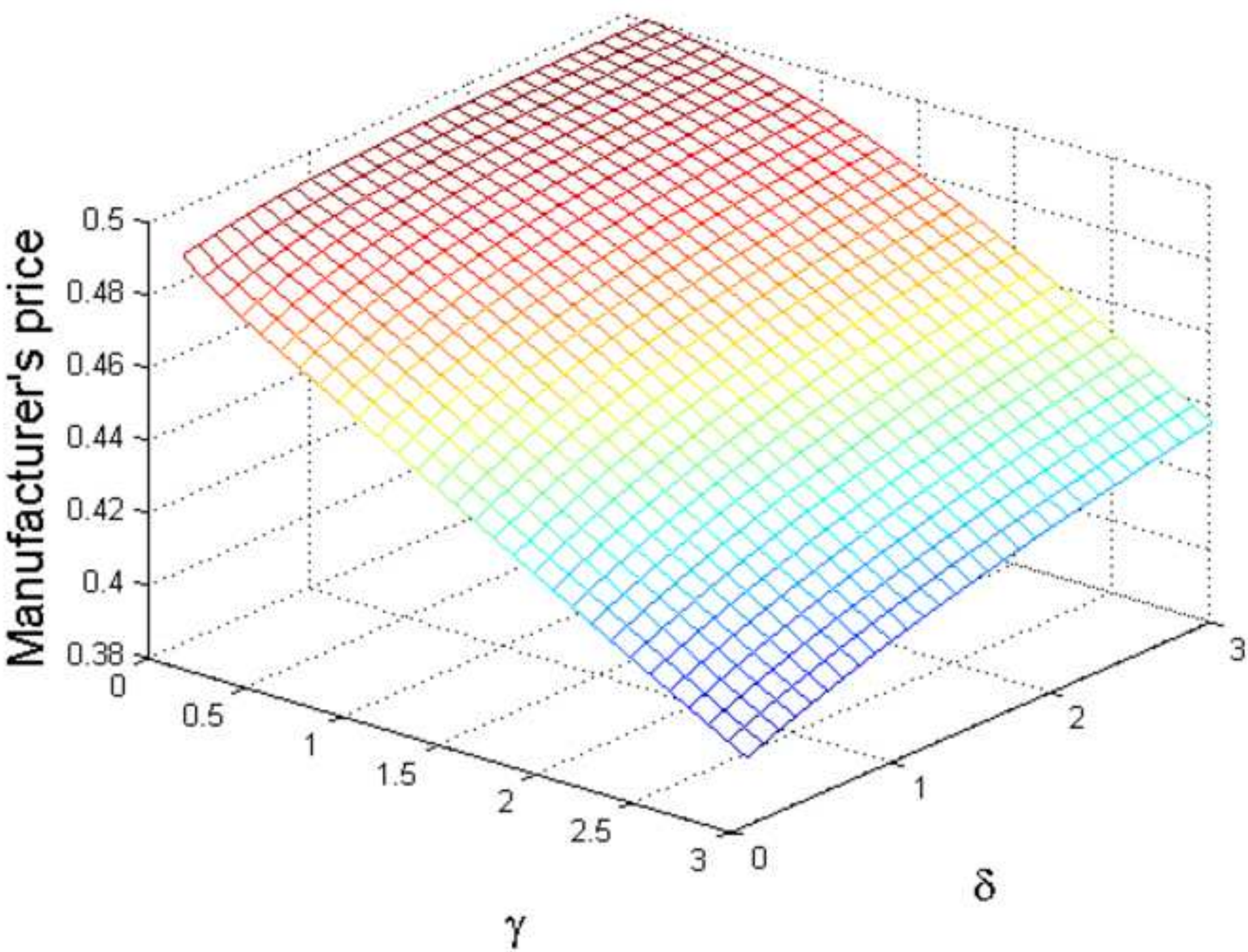
Zhang, J., & Chen, J. (2013). Coordination of information sharing in a supply chain. International Journal of Production Economics, 143(1), 178-187.

Zhao, L., Zhang, J., & Xie, J. (2015). Impact of demand price elasticity on advantages of cooperative advertising in a two-tier supply chain. International Journal of Production Research, 1-11.

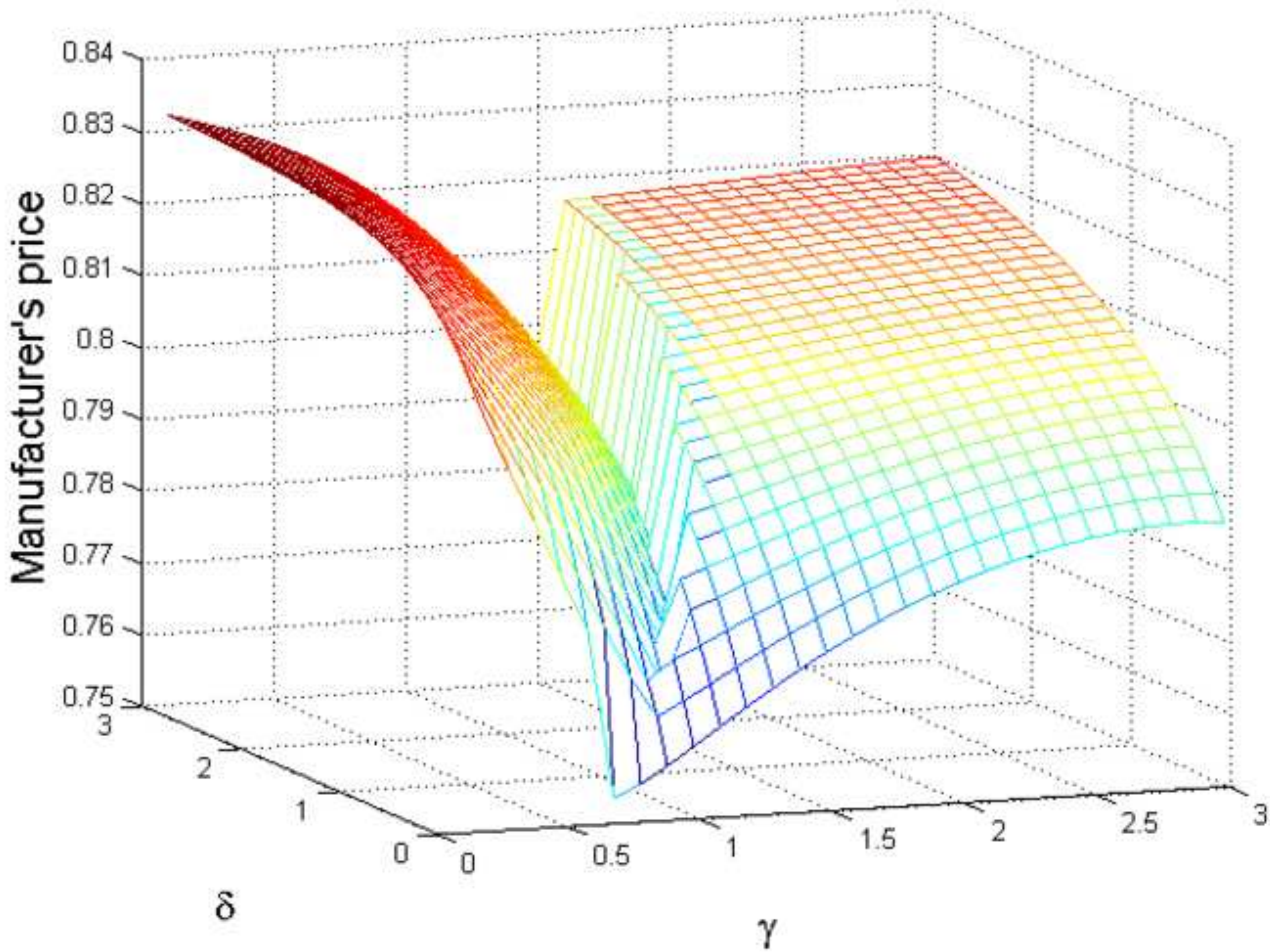

Retailer margin for ν = 1
0.5
0.495
0.49
0.485
0.48
0.475
0.47
0.465
0.46
0.455
3
2.5
2
1.5
1
0.5
0
δ
0
1
2
3
γ



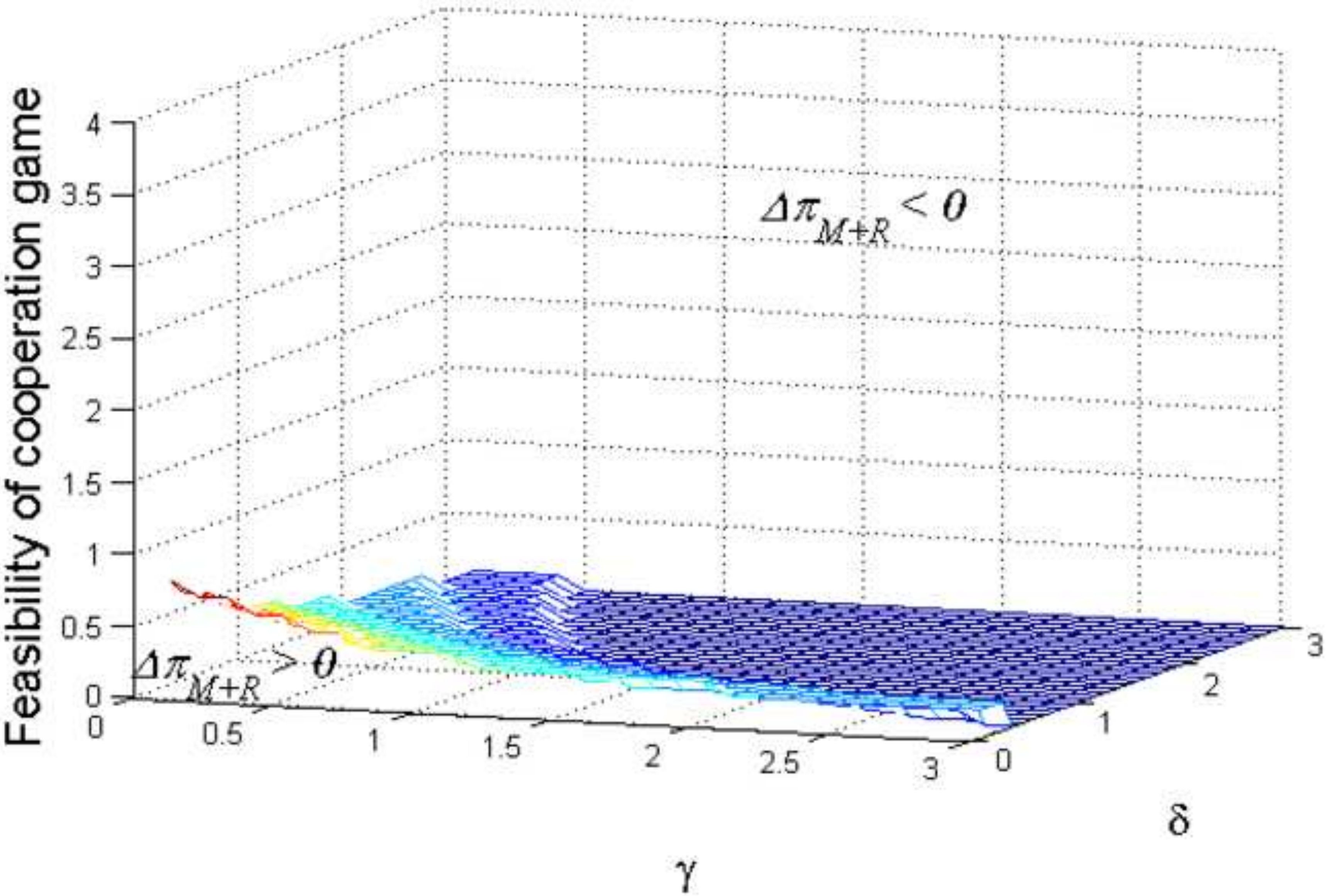
0.7
0.65
0.6
0.55
0.5
0.45
Retailer margin
↑ν = 2
↑ν = 1.5
↑ν = 1
3
2
1
0
δ
0
1
2
3
γ



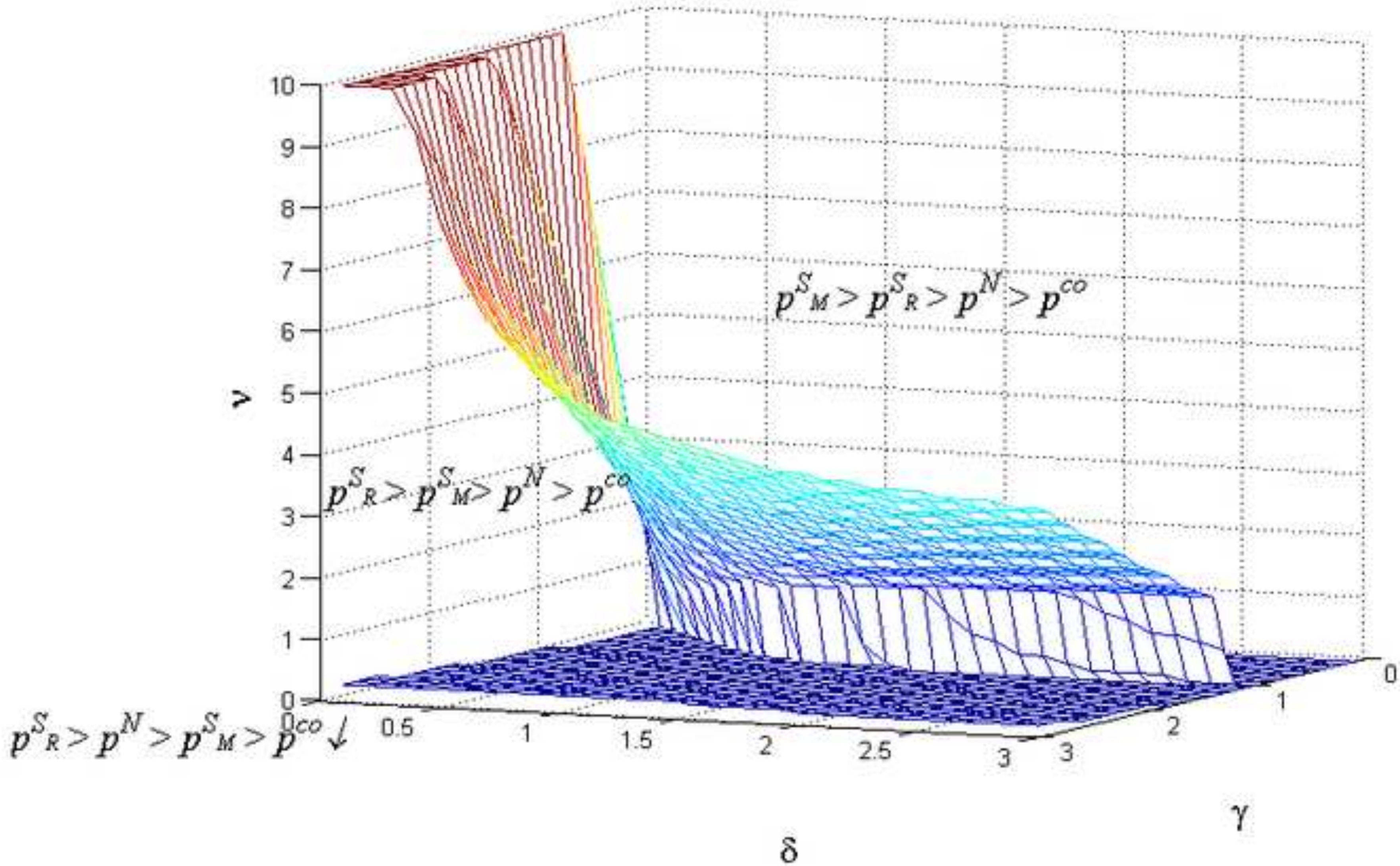
Manufacturer's price
0.5
0.48
0.46
0.44
0.42
0.4
0.38
0
0.5
1
1.5
2
2.5
3
γ
0
1
2
3
δ



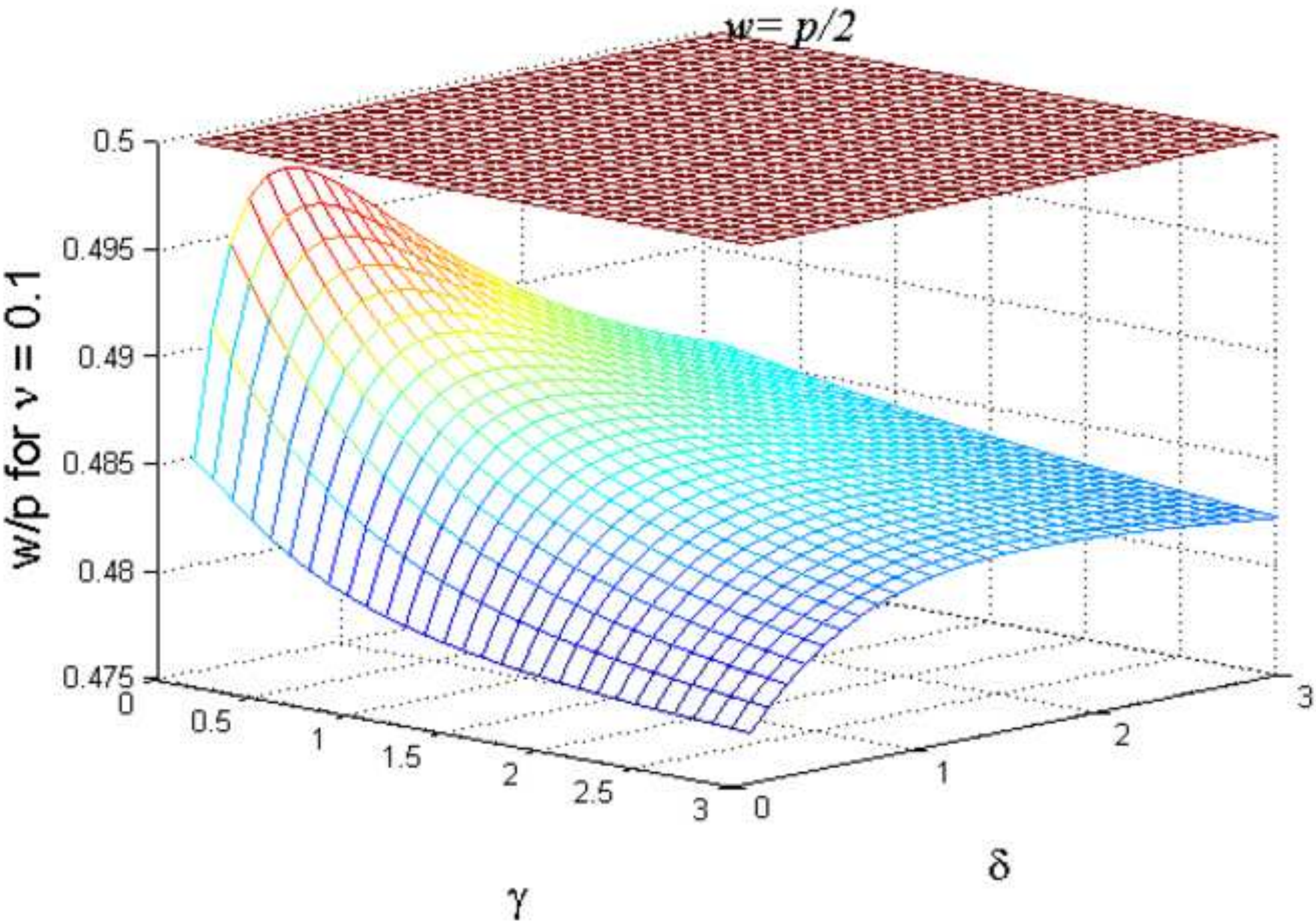
Manufacturer's price
0.84
0.83
0.82
0.81
0.8
0.79
0.78
0.77
0.76
0.75
3
2
1
0
δ
0
0.5
1
1.5
2
2.5
3
γ



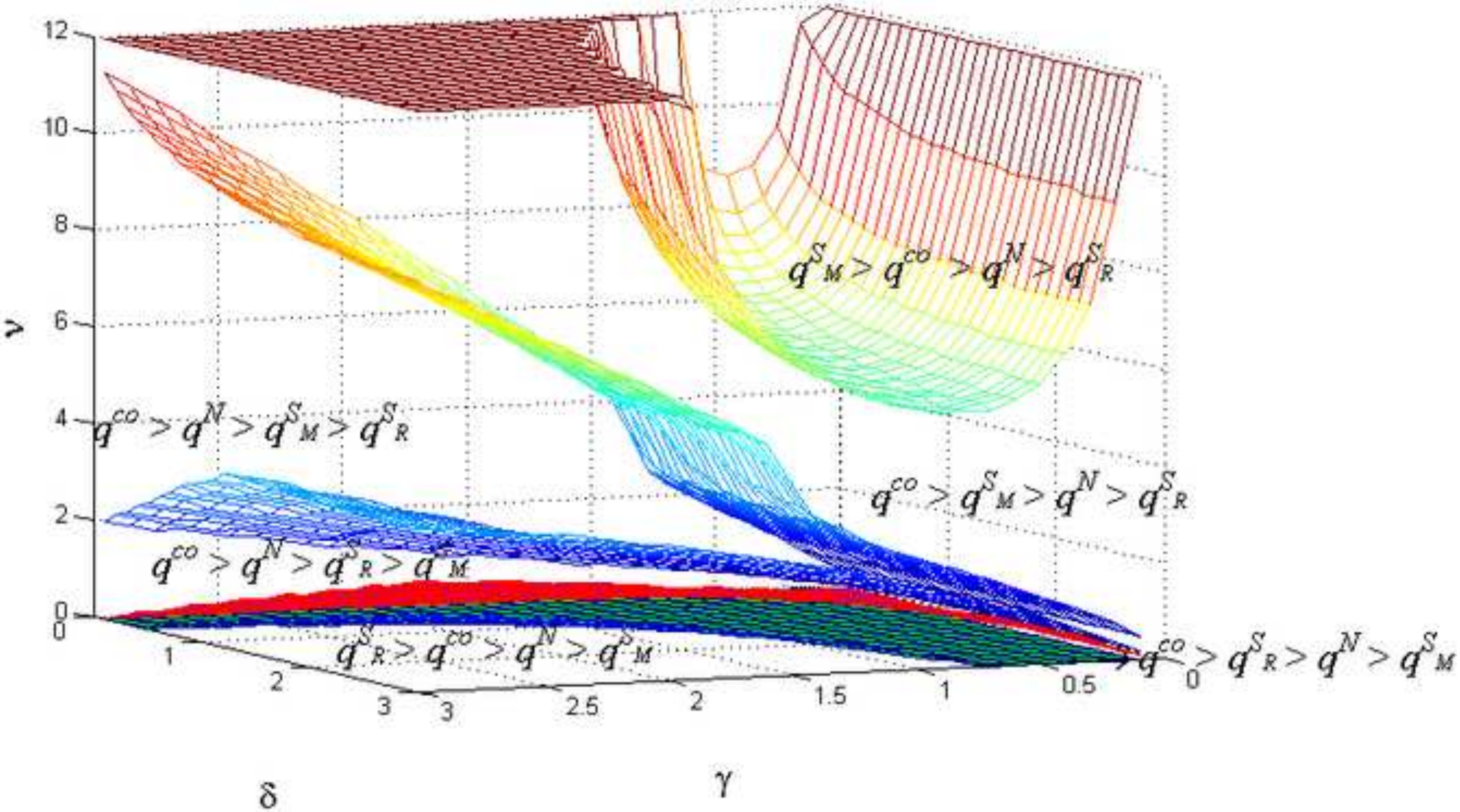
Feasibility of cooperation game
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3
0
1
2
3
$\Delta\pi_{M+R} < 0$
$\Delta\pi_{M+R} > 0$
δ
γ



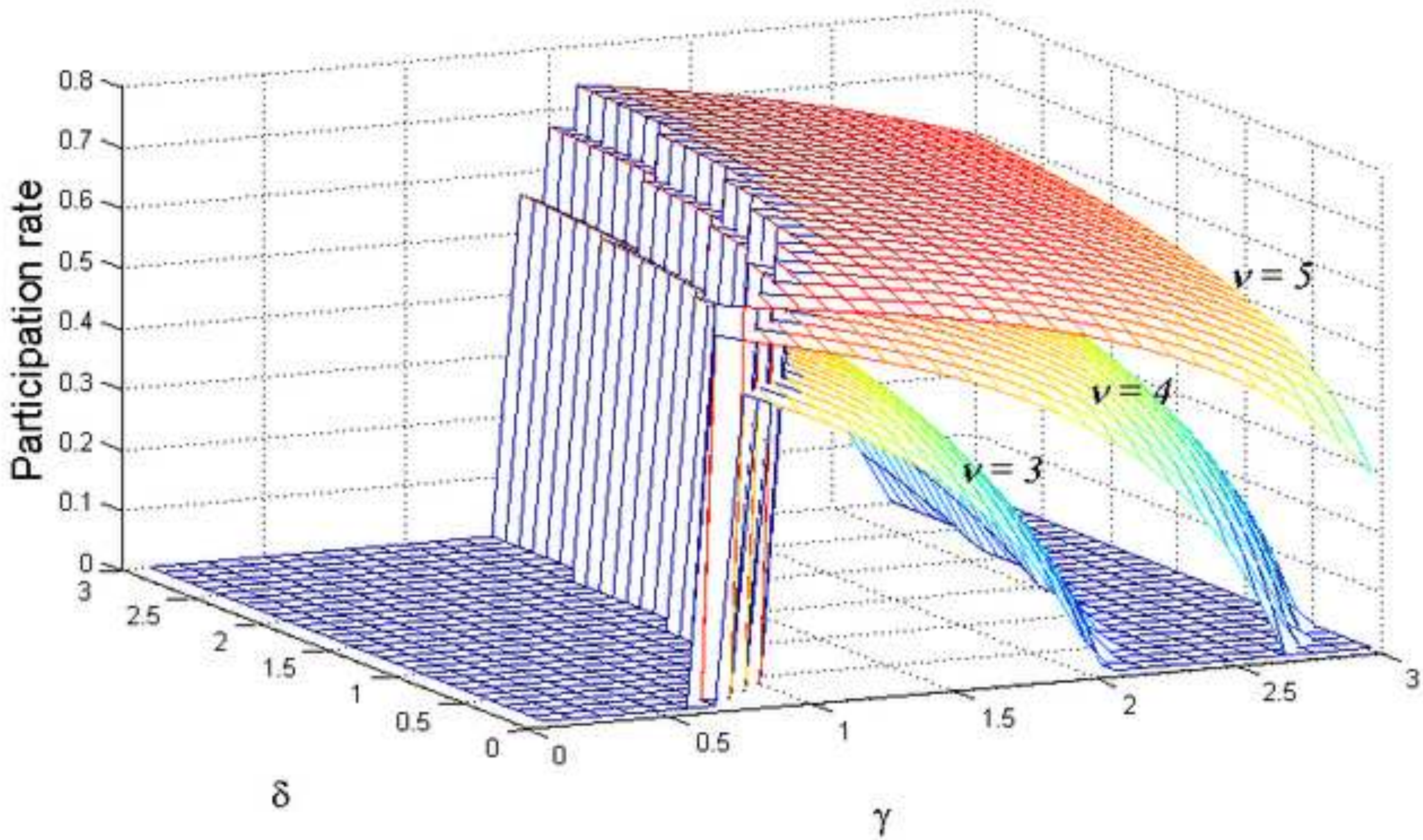
10
9
8
7
6
v
5
4
3
2
1
0
$p^{S}{}_{M} > p^{S}{}_{R} > p^{N} > p^{co}$
$p^{S}{}_{R} > p^{S}{}_{M} > p^{N} > p^{co}$
$p^{S}{}_{R} > p^{N} > p^{S}{}_{M} > p^{co}$
0.5
1
1.5
2
2.5
3
0
1
2
3
γ
δ



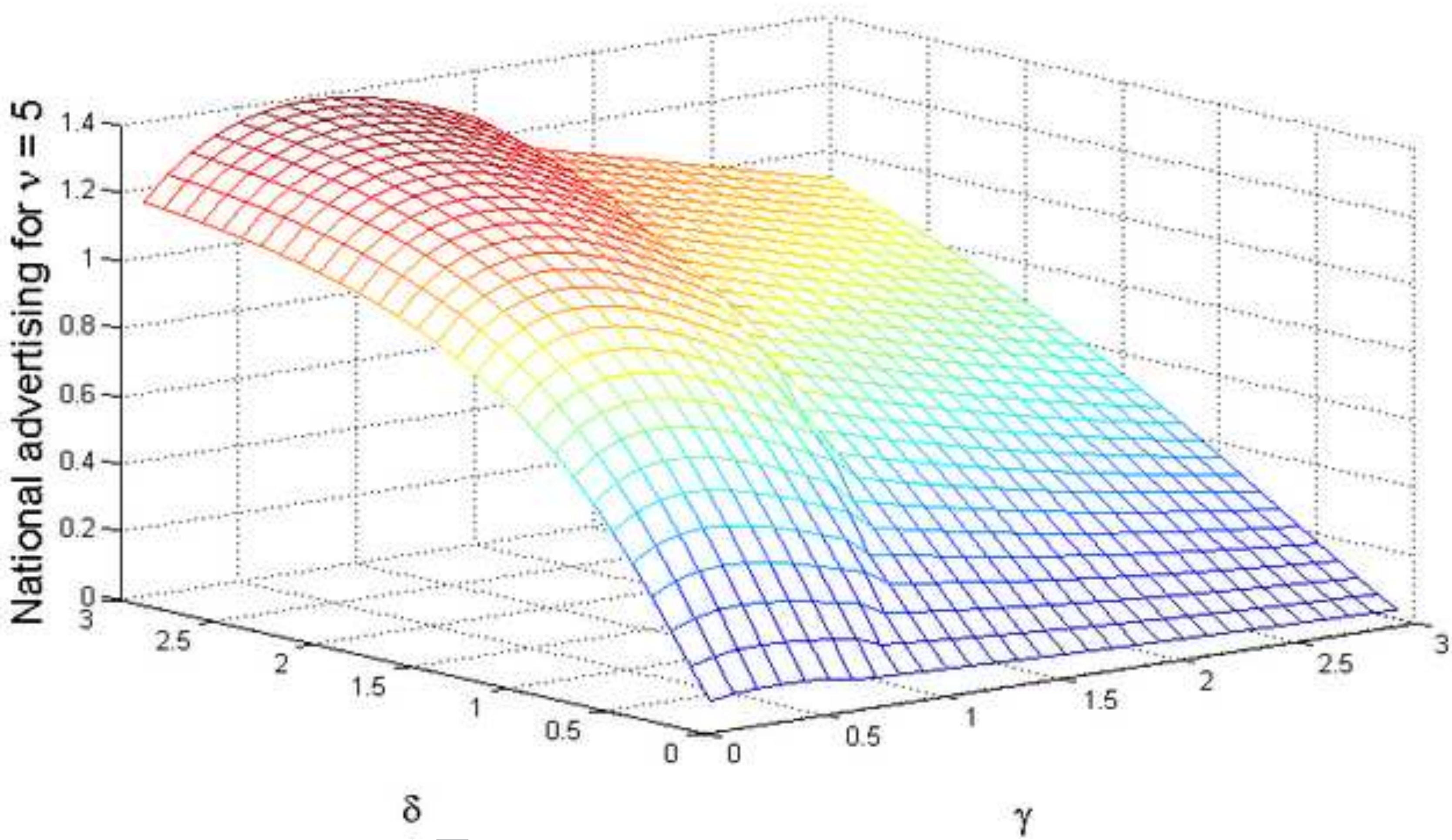
w= p/2
w/p for ν = 0.1
0.5
0.495
0.49
0.485
0.48
0.475
0
0.5
1
1.5
2
2.5
3
γ
0
1
2
3
δ



$q^{S}_{M} > q^{co} > q^{N} > q^{S}_{R}$
$q^{co} > q^{N} > q^{S}_{M} > q^{S}_{R}$
$q^{co} > q^{S}_{M} > q^{N} > q^{S}_{R}$
$q^{co} > q^{N} > q^{S}_{R} > q^{S}_{M}$
$q^{S}_{R} > q^{co} > q^{N} > q^{S}_{M}$
$q^{co} > q^{S}_{R} > q^{N} > q^{S}_{M}$
ν
δ
γ



Participation rate
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$\nu = 5$
$\nu = 4$
$\nu = 3$
3
2.5
2
1.5
1
0.5
0
δ
0
0.5
1
1.5
2
2.5
3
γ



National advertising for ν = 5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
δ
3
2.5
2
1.5
1
0.5
0
γ
0
0.5
1
1.5
2
2.5
3

Highlights

- Supply chain coordination via cooperative advertising and pricing is investigated.
- A relatively general demand function in a manufacturer-retailer channel is proposed.
- The cooperation game is strongly found to be infeasible.
- The manufacturer does not contribute in inefficient local advertising.
- Manufacturer’s margin is always lower than retailer’s in Stackelberg retailer game.