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Pricing, inventory and production policies in a supply chain of pharmacological products with rework process: a game theoretic approach

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Abstract In this paper, an economic production quantity model in a three levels supply chain including multiple non-competing suppliers, single manufacturer and multiple non-competing retailers for multiple products with rework process under integrated and non-integrated structures is developed. In this chain, suppliers ship raw material to the manufacturer and the manufacturer combines the fixed percentage of different kinds of raw material to produce the finished products and then delivers them to the retailers. The Stackelberg game is established among the members of the chain. Optimizing the total profit of the chain under both structures applying the optimal pricing, inventory and production policies is the purpose of the research. Eventually, a numerical example is presented for comparing the total profit of the integrated and non-integrated chains of proposed model and some sensitivity analyses are performed to study the effects of parameter changes on the decision variables and the chain profit.

Keywords Pricing · Inventory control · Production · Health care · Rework process · Game theory · Supply chain

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1 Introduction and literature review

Nowadays, supply chain management has been widely concerned by organizations and companies. In this management method, all components and parts, which are took part in each chain, intend to provide products or services for the customers. Actually, supply chains as the procurement and distribution networks of products play the crucial and principal role in the market. The main goal of each chain, comprised several enterprises, is to optimize the total profit of chain under satisfying the market demand in order to attract the customers. Furthermore, due to the high competition, the strategic, tactical and operational policies such as pricing, ordering and manufacturing policies are applied to improve the efficiency and effectiveness of entire supply chain against the competing chains. Recently, studying the optimal decision policies in the area of supply chain has been investigated by many researchers.

For instance, Dada and Srikanth (1987) analyzed seller's decision to suggest the quantity discount to the buyer. Abad (2003) investigated pricing and lot sizing decisions for a perishable product when partial backordering is allowed. Cárdenas-Barrón (2007) offered an inventory model in an n-stage multiple customers supply chain where a company supplies products to the several customers. Rosenthal (2008) studied the problem of determining prices in an integrated supply chain. Transchel and Minner (2009) investigated the effect of dynamic pricing on the single product economic order decision of a monopolist retailer. Xiao et al. (2010) employed ordering, lead time and wholesale pricing decisions in a three layers supply chain for a deterioration item. Thangam and Uthayakumar (2010) developed an economic order quantity based model for perishable products under retailer's partial trade credit policy and price dependent demand with two-storage facility to model a profit maximization problem. Soon (2011) presented a general review of pricing models for multiple items. Parthasarathi et al. (2011) searched the role of quantity discounts and return policies under a stock dependent price-sensitive newsvendor framework to analyze the coordination of supply chain in which manufacturer offers quantity discounts and return policies to two competing-retailers who face uncertain, price-sensitive and stock dependent demand. Pal et al. (2012a) characterized the optimal ordering decision of an integrated three layers supply chain including multiple distributors, one manufacturer and multiple non-competing retailers a production-inventory model for multi-item to minimize the chain costs. Hua et al. (2012) determined the ordering and pricing decisions where the distributor offers free shipping. Cárdenas-Barrón et al. (2012a) presented an enhanced algorithm to solve a vendor managed inventory (VMI) system on an economic order quantity (EOQ) model with two types of linear and fixed backorders costs for multiple products with several constraints. Also a promoted algorithm to obtain a maximum profit or minimum cost of a three layers supply chain on an economic production quantity (EPQ) model is proposed by Cárdenas-Barrón et al. (2012b). Lim (2013) considered a robust optimization model to determine both of the bundling price and order quantity in a two stages supply chain for a retailer under uncertainty of demand's parameters and purchase cost. Mutlu and Cetinkaya

(2013) studied a carrier-retailer channel and optimized profit of decentralized and centralized channels where demand is price sensitive and each member of channel sets own price. Jonrinaldi and Zhang (2013) suggested a model for coordinating integrated production and inventory cycles for multiple products in a supply chain including two-level distributors, one manufacturer and multi-retailer under finite horizon period. Ben-Daya et al. (2013) considered the joint economic lot sizing problem in a three layers supply chain where single distributor, single manufacturer and multiple retailers are the members of the chain. A heuristic algorithm to solve an economic production quantity (EPQ) vendor–buyer integrated model with a budget constraint and also just in time (JIT) approach is proposed by Cárdenas-Barrón et al. (2014) which involves less expensive and computation than previous ones. Taleizadeh and Noori-daryan (2014) studied pricing, ordering and manufacturing decisions in a three layers supply chain where a supplier, a manufacturer and some non-competing retailers are the chain members.

Game theory usually is utilized as a reliable solution method for solving the problem occurred in supply chains. Migdalas (2002) surveyed the raising growth of game theory approach in important fields of managerial accounting and finance. Esmaeili et al. (2009) offered multiple seller-buyer models by considering cooperation and competition between seller and buyer. Leng and Parlar (2010) developed a supply chain with multiple distributors and one manufacturer where the distributors produce component of product and the manufacturer assemble them. Seyed-Esfahani et al. (2011) considered vertical cooperative advertising and pricing policies in a supply chain including single manufacturer and single retailer when demand is price and advertisement sensitive. Zhang and Liu (2013) analyzed a three-level green supply chain in which market demand correlates with item green degree and game theory is employed to solve four proposed models. He (2013) presented that how firms sequentially can make price and quantity decisions under demand and supply risks. Taleizadeh and Noori-daryan (2014) also employed the Stackelberg game-theoretic approach to solve their model.

Rework process is one of the serious issues to which companies are faced, because when the system is in out of control state, the defective items are generated during the production process and the companies had to pay incremental expenditure and time to remanufacture them to healthy products. From 1982 many researchers paid more attentions to this topic. As the first study, Rosenblatt and Lee (1986) studied the impacts of an imperfect production process on production cycle time and during the production process. Cheng (1991) studied the classical economic order quantity (EOQ) model in which product quality is not perfect. Chiu (2003) focused on an economic production quantity (EPQ) model and studied the effects of reworking of defective items when shortage is permitted. Chiu et al. (2004) analyzed the EPQ model with imperfect rework process and random defective rate. They assumed that all generated items will be inspected, the defective rate is a random variable and the reworking of defective items starts after finishing the production process. Jamal et al. (2004) studied a production system generating defective products and they determined the optimum batch quantity in single-stage manufacturing system where the rework is performed under two different operational policies to optimize the total cost of system. Cárdenas-Barrón (2008) presented a simple derivation of two inventory policies of Jamal et al. (2004). Taleizadeh et al. (2010a) studied an EPQ model with scraped item under multi product single machine manufacturing system. Then Taleizadeh et al. (2010b) developed their previous work by adding partial backordering and service level constraint, but rework was not considered. Taleizadeh et al. (2010c) developed a multi product single machine economic production quantity model with service level constraint, random defective rate, partial back-ordering and rework process in which the cycle length, production quantity and back ordered quantity of each product are the decision variables. Sana (2011) investigated an integrated production and inventory model for distributor, manufacturer and retailer in a supply chain with perfect and imperfect quality items. Taleizadeh et al. (2011a) proposed an EOQ model for a multi-product inventory system in which those are produced on a single machine with a limited capacity. Taleizadeh et al. (2011b) suggested an EPQ model for multi-product single-machine manufacturing systems with rework process and back-ordering. Taleizadeh et al. (2012) developed an EPQ model for a multi-product single stage production system with immediate rework process where is different from Taleizadeh et al. (2011a, b)'s research. Pal et al. (2012b) extended an integrated production and inventory model of three layers supply chain including one distributor, one manufacturer and one retailer for reworkable items where distributor delivers raw material to the manufacturer and manufacturer sends back the defective raw material after inspection with less market price to the distributors. Pal et al. (2013) developed an EPQ model for an imperfect production system in which the defective products generated in out of control position of manufacturing system and reworked after common production time. Chakraborty and Giri (2014) considered the integrated effects of imperfect preventive maintenance (PM), imperfect rework of defective products, inspections and shift on the optimal decisions for a deteriorating production system.

Based on the above literature, the optimal decision strategies are incorporated in optimizing the costs of supply chains from many years ago (Xiao et al. 2010; Hua et al. 2012; Pal et al. 2012a; Ben-Daya et al. 2013; Taleizadeh and Noori-daryan 2014) and also there are many researches considered the inevitable rework process which results in the additional costs during the production process to manage the time and costs of production particularly in supply chains which leads to take more time in order to respond customers' demand and sometimes backorder the market demand (Sana 2011; Taleizadeh et al. 2011a, b, 2012; Pal et al. 2012b, 2013). In addition, many additional researches are related to the explained research areas which are not presented here.

But an integrated pricing, manufacturing and inventory control model in a multi-level supply chain with rework process as an interesting issue is considered by none of them discussed. So, in this research, an economic production quantity (EPQ) model in a three-layer supply chain is developed employing joint pricing, ordering and manufacturing strategies where multiple non-competing suppliers, a manufacturer and several non-competing retailers compose the members of the chain for multiple pharmacological raw materials and multi-product under rework process which is prevalent and often necessary process in mass industrial production which is not broached at least in health care area.

As mentioned in advance, recently, Pal et al. (2012a) developed a production and inventory model in an integrated three layers supply chain involving multi-supplier, one manufacturer and multi-retailer applying the optimal just ordering policy to minimize the chain costs. Here, we developed Pal et al. (2012a)'s research applying the combinations of optimal pricing and ordering decision strategies to maximize the profit of the multi-level chain with rework process under both integrated and non-integrated structures of proposed supply chain. Additionally, this research enjoys the real case of pharmacological products produced in an international level and also are well known articulated in the definition of problem section.

The decision variables of problem are the order quantities of non-competing suppliers and the selling prices of the manufacturer and the non-competing retailers. The rest of paper is organized as follows. The problem description is presented in Sect. 2. The mathematical model is shown in Sect. 3. The solution method is defined in Sect. 4. In Sect. 5 a numerical example and sensitivity analysis are presented and Sect. 6 contains conclusion.

2 Problem description

Consider a three-layer supply chain contains multi-supplier, one manufacturer and multi-retailers where the suppliers receive raw material from external suppliers (outside of the chain) and each supplier delivers the one kind of raw material to the manufacturer. Then the manufacturer combines the certain percentage of various kinds of raw material to produce products and ships them to the non-competing

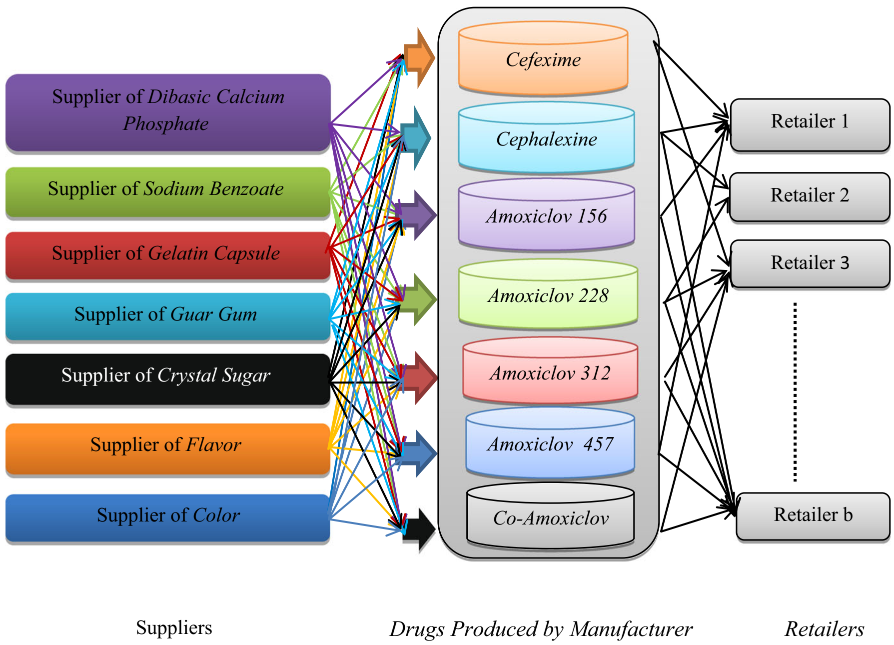

Fig. 1 A three layers supply chain

retailers according to their different order quantities. The structure of this chain is shown in Fig. 1. This problem is defined for a real case to which we have faced in a pharmacological industry.

According to our experience in this manufacturing system, during the production process, defective items are generated but all of them are reworkable and will be reworked after finishing the regular production process, so in this research is supposed that the rework process is done after finishing the production process and then the health products are launched to the market. In this paper, our aim is to determine the quantity orders of suppliers, the selling prices of the manufacturer and the retailers for each product such that the total benefit is maximized. Other assumptions which are defined are as below.

1. Demand is price sensitive.
2. The holding costs and ordering costs for the members of each stage are different.
3. Multiple raw materials and multiple products are assumed.
4. Production rate is bigger than retailers' demand rates.
5. Production and reworking process are performed at the same production rate.
6. Proportion of defective items for each product is constant.
7. Both integrated and non-integrated structures of supply chain are considered.
8. Shortage is not allowed.
9. All the parameters of this model are deterministic.

To model the problem the following notations are used.

d_k^s	The manufacturer's demand for the kth raw material
q_k	The quantity order of kth supplier to external supplier (outside of chain)
h_k^s	The holding cost of kth raw material per unit per unit time for supplier
A_k^s	The ordering cost of kth raw material per order for supplier
HC_k^s	The total holding cost of kth raw material per cycle for supplier
OC_k^s	The total ordering cost of kth raw material per cycle for supplier
C_k^s	The unit purchasing cost of kth raw material for supplier
S_k^s	The unit selling price of kth raw material to the manufacturer under non-integrated structure
T_k^s	The cycle length of kth supplier
λ_k^s	The percentage of kth raw material for producing ith product, $0 \leq \lambda_{ik} \leq 1$, $\sum_{i=1}^{m} \lambda_{ik} = 1$
d_i^m	The demand of retailers for ith product, $d_{ji}^m = a - bS_i^m$ where a, b are positive coefficients
P_i	The production rate of ith product
h_i^m	The holding cost of ith product per unit per unit time for manufacturer
A_i^m	The setup cost of ith product for manufacturer
HC_i^m	The total holding cost of ith product per cycle for manufacturer
OC_i^m	The total setup cost of ith product for manufacturer
$C(P_i)$	The production cost of manufacturer for ith product, $C(P_i) = F_i/P_i + v_i P_i$
F_i	The fixed cost of production for ith product

v_i	The variation cost of production for ith product
S_i^m	The selling price of manufacturer for ith product under non-integrated structure
T_i^1	The cycle length of production up-time for ith product
T_i^2	The cycle length of rework for ith product
T_i^3	The cycle length of production down-time for ith product
T_i^m	The cycle length of manufacturer for the ith product
ρ_{ji}	The percentage of demand of ith product for satisfying demand of jth retailer $0 < \rho_{ji} \leq 1$, $\sum_{j=1}^{b} \rho_{ji} = 1$
α_i	The proportion of defectives of ith product, $0 < \alpha_i < 1$
d_{ji}^r	The demand of ith product to which jth retailer is faced $d_{ji}^r = a - bS_{ji}^r$
h_{ji}^r	The unit holding cost of jth retailer for ith product per unit per unit time
A_{ji}^r	The ordering cost of jth retailer for ith product per order
HC_{ji}^r	The total holding cost of jth retailer for ith product per cycle
OC_{ji}^r	The total ordering cost of jth retailer for ith product per cycle
S_{ji}^r	The selling price of jth retailer for ith product under non-integrated structure
S_i^r	The selling price for ith product under integrated structure
T_{ji}^1	The cycle length of jth retailer for receiving and selling ith product
T_{ji}^2	The cycle length of jth retailer for only selling ith product
T_{ji}^r	The cycle length of jth retailer for ith product
TP_k^s	The total profit of kth supplier
TP^m	The total profit of manufacturer
TP_j^r	The total profit of jth retailer
TP	The total profit of supply chain

3 Mathematical model

In this section, the total profit of each stage is formulated which is obtained from subtraction of its total cost from its revenue.

3.1 Suppliers' model

The suppliers receive raw material from external suppliers and send them to the manufacturer. Since each supplier sales one kind of raw material, the number of suppliers is equal to the number of raw materials. We assume that kth supplier supplies kth raw material and send that to the manufacture at a rate of d_k^s. Then, the differential equation of inventory level of kth supplier is:

$$\frac{dI_k^s(t)}{dt} = -d_k^s, \quad 0 \leq t \leq T_k^s, \; k = 1, 2, \ldots, n \tag{1}$$

According to Fig. 2, we have:

$$I_k^s(0) = q_k \text{ and } I_k^s(T_k^s) = 0$$

Therefore, the kth supplier's inventory level is:

$$I_k^s(t) = q_k - d_k^s t, \quad 0 \le t \le T_k^s \tag{2}$$

Based on the boundary condition, we have:

$$I_k^s(T_k^s) = 0 \Rightarrow T_k^s = \frac{q_k}{d_k^s} \tag{3}$$

Since the total profit of kth supplier is equal to subtraction of the holding and ordering costs of kth raw material from his revenue, the total holding cost of kth supplier is:

$$HC_k^s = \frac{1}{T_k^s}\left[h_k^s \int_0^{T_k^s} (q_k - d_k^s t)dt\right] = h_k^s q_k - h_k^s\left(\frac{d_k^s T_k^s}{2}\right) = \frac{h_k^s q_k}{2} \tag{4}$$

And the ordering cost is:

$$OC_k^s = \frac{A_k^s}{T_k^s} = \frac{d_k^s A_k^s}{q_k} \tag{5}$$

Hence, the total profit of kth supplier is:

$$TP_k^s(q_k) = (S_k^s - C_k^s)d_k^s - \left(\frac{h_k^s q_k}{2} + \frac{d_k^s A_k^s}{q_k}\right) \tag{6}$$

3.2 Manufacturer's model

The manufacturer receives raw material from the suppliers to produce various kinds of products which are produced under different combination of different types of raw material during the production process. We assumed that during production process defective items are produced at the rate of $\alpha_i P_i$ but after finishing the regular production process, defective items will be reworked at the rate of P_i which is:

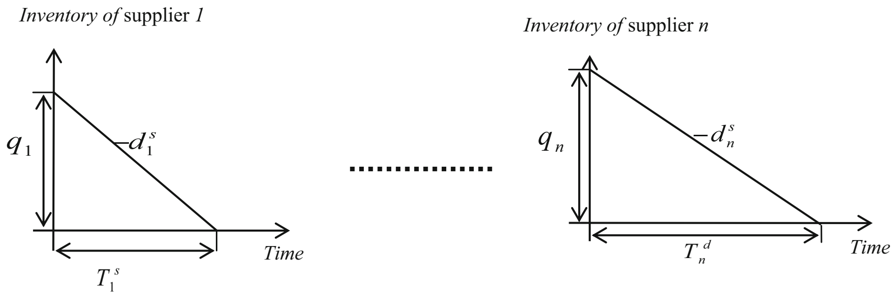

Fig. 2 Suppliers' inventory level

$$P_i = \sum_{k=1}^{n} \lambda_{ik} d_k^s \tag{7}$$

According to Fig. 3, the manufacturer's cycle time is equal to summation of production run-time of ith product (T_i^1), the reworking time of ith product (T_i^2) and the production downtime of ith product (T_i^3), for $i = 1, 2,\ldots, m$, which are:

$$T_i^1 = \frac{\sum_{k=1}^{n} \lambda_{ik} q_k}{P_i} \tag{8}$$

$$T_i^2 = \alpha_i \frac{\sum_{k=1}^{n} \lambda_{ik} q_k}{P_i} \tag{9}$$

$$T_i^3 = \frac{\left(1 - (1 + \alpha_i)\frac{d_i^m}{P_i}\right) \sum_{k=1}^{n} \lambda_{ik} q_k}{d_i^m} \tag{10}$$

Hence, the cycle length of the manufacturer is:

$$T_i^m = \sum_{h=1}^{3} T_i^h = \frac{\sum_{k=1}^{n} \lambda_{ik} q_k}{d_i^m} \tag{11}$$

Based on Fig. 3, the differential equations of these inventory levels are:

$$\frac{dI_i^m}{dt} = (1 - \alpha_i) P_i - d_i^m, \quad 0 \le t \le T_i^1 \tag{12}$$

$$\frac{dI_i^m}{dt} = P_i - d_i^m, \quad T_i^1 \le t \le T_i^1 + T_i^2 \tag{13}$$

$$\frac{dI_i^m}{dt} = -d_i^m, \quad T_i^1 + T_i^2 \le t \le T_i^1 + T_i^2 + T_i^3 = T_i^m \tag{14}$$

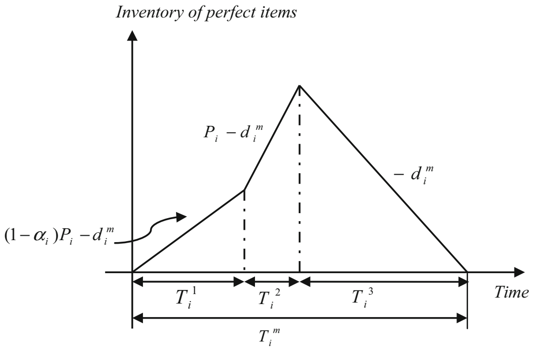

Fig. 3 Manufacturer's inventory level

Using the boundary conditions, the manufacturer's inventory level of ith product is:

$$I_i^m = \left((1-\alpha_i)P_i - d_i^m\right)t, \quad 0 \le t \le T_i^1 \tag{15}$$

$$I_i^m = \left((1-\alpha_i)P_i - d_i^m\right)T_i^1 + \left(P_i - d_i^m\right)\left(t - T_i^1\right), \quad T_i^1 \le t \le T_i^1 + T_i^2 \tag{16}$$

$$I_i^m = d_i^m\left(T_i^m - t\right), \quad T_i^1 + T_i^2 \le t \le T_i^m \tag{17}$$

Using Eqs. (8)–(11), the total holding cost of manufacturer is:

$$\begin{aligned} HC_i^m =& \frac{1}{T_i^m}\left[h_i^m\left[\int_0^{T_i^1}\left((1-\alpha_i)P_i - d_i^m\right)tdt \right.\right.\\ &\left.\left.+\int_{T_i^1}^{T_i^1+T_i^2}\left(\left(P_i - d_i^m\right)t - \alpha_i P_i T_i^1\right)dt + \int_{T_i^1+T_i^2}^{T_i^m} d_i^m\left(T_i^m - t\right)dt\right]\right]\\ =&\frac{h_i^m\sum_{k=1}^n \lambda_{ik}q_k}{2}\left[\frac{\left(1+\alpha_i+\alpha_i^2\right)d_i^m}{P_i}\right] \end{aligned} \tag{18}$$

And his ordering cost is:

$$OC_i^m = \frac{A_i^m}{T_i^m} = \frac{d_i^m A_i^m}{\sum_{k=1}^n \lambda_{ik}q_k} \tag{19}$$

So, the total profit of manufacturer is:

$$\begin{aligned} TP^m(S_i^m) =& \sum_{i=1}^m\left(S_i^m - C(P_i) - \sum_{k=1}^m S_k^s\right)d_i^m - \left(HC_i^m + OC_i^m\right)\\ =&\sum_{i=1}^m\left(S_i^m - C(P_i) - \sum_{k=1}^m S_k^s\right)(a - bS_i^m)\\ &-\left[\frac{h_i^m\sum_{k=1}^n \lambda_{ik}q_k}{2}\left[1+\frac{\left(1+\alpha_i+\alpha_i^2\right)(a-bS_i^m)}{P_i}\right] + \frac{(a-bS_i^m)A_i^m}{\sum_{k=1}^n \lambda_{ik}q_k}\right] \end{aligned} \tag{20}$$

3.3 Retailers' model

Retailers order different types of products to the manufacturer and ith product inventory level of jth retailer during T_{ji}^1 increase at the rate of $\rho_{ji}d_i^m - d_{ji}^r$ and during T_{ji}^2 decreases at the rate of d_{ji}^r. According to Fig. 4, the cycle length of jth retailer is obtained from summation of T_{ji}^1 and T_{ji}^2 of ith product, for $i = 1, 2,\ldots, m$, which is:

$$T_{ji}^1 = \frac{\sum_{k=1}^n \lambda_{ik}q_k}{d_i^m} \tag{21}$$

$$T_{ji}^2 = \frac{\left(\rho_{ji} - \frac{d_{ji}^r}{d_i^m}\right)\sum_{k=1}^n \lambda_{ik}q_k}{d_{ji}^r} \tag{22}$$

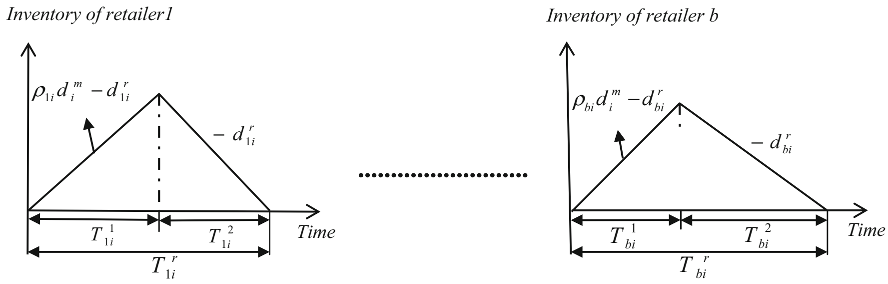

Fig. 4 Retailers' inventory level

The cycle time of jth retailer is:

$$T_{ji}^r = \sum_{x=1}^{2} T_{ji}^x = \frac{\rho_{ji} \sum_{k=1}^{n} \lambda_{ik} q_k}{d_{ji}^r} \tag{23}$$

The related differential equations of these levels are:

$$\frac{dI_{ji}^r}{dt} = \rho_{ji} d_i^m - d_{ji}^r, \quad 0 \le t \le T_{ji}^1, \quad j = 1, 2, \ldots, b \tag{24}$$

$$\frac{dI_{ji}^r}{dt} = -d_{ji}^r, \quad T_{ji}^1 \le t \le T_{ji}^1 + T_{ji}^2 = T_{ji}^r, \quad j = 1, 2, \ldots, b \tag{25}$$

It is evident from Fig. 4 that:

$$I_{ji}^r(t) = \left(\rho_{ji} d_i^m - d_{ji}^r\right) t, \quad 0 \le t \le T_{ji}^1 \tag{26}$$

$$I_{ji}^r(t) = d_{ji}^r (T_{ji}^r - t), \quad T_{ji}^1 \le t \le T_{ji}^r \tag{27}$$

The total holding cost of jth retailer for ith product is:

$$\begin{aligned} HC_{ji}^r &= \frac{1}{T_{ji}^r} \left[h_{ji}^r \left[\int_0^{T_{ji}^1} \left(\rho_{ji} d_i^m - d_{ji}^r\right) t\,dt + \int_{T_{ji}^1}^{T_{ji}^r} d_{ji}^r \left(T_{ji}^r - t\right) dt \right] \right] \\ &= \frac{1}{T_{ji}^r} \left[h_{ji}^r \left[\frac{1}{2} \rho_{ji} d_i^m (T_{ji}^1)^2 + \frac{1}{2} d_{ji}^r (T_{ji}^r)^2 - d_{ji}^r T_{ji}^1 T_{ji}^r \right] \right] = \frac{h_{ji}^r \rho_{ji} \sum_{k=1}^{n} \lambda_{ik} q_k}{2} \left[1 - \frac{d_{ji}^r}{\rho_{ji} d_i^m} \right] \end{aligned} \tag{28}$$

And ordering cost is equal to:

$$OC_{ji}^r = \frac{A_{ji}^r}{T_{ji}^r} = \frac{d_{ji}^r A_{ji}^r}{\rho_{ji} \sum_{k=1}^{n} \lambda_{ik} q_k} \tag{29}$$

By employing Eqs. (21) to (23), the total profit of jth retailer is:

$$\begin{aligned} TP_j^r &= \sum_{i=1}^{m}\left(S_{ji}^r - S_i^m\right)d_{ji}^r - \left(HC_{ji}^r + OC_{ji}^r\right) \\ &= \sum_{i=1}^{m}\left(S_{ji}^r - S_i^m\right)(a - bS_{ji}^r) - \left(\frac{h_{ji}^r\rho_{ji}\sum_{k=1}^{n}\lambda_{ik}q_k}{2}\left[1 - \frac{(a - bS_{ji}^r)}{\rho_{ji}(a - bS_i^m)}\right] + \frac{(a - bS_{ji}^r)A_{ji}^r}{\rho_{ji}\sum_{k=1}^{n}\lambda_{ik}q_k}\right) \end{aligned} \tag{30}$$

Therefore, the total profit of supply chain resulted from summation of the profit functions of the chain members is:

$$TP = \sum_{k=1}^{n} TP_k^s + TP^m + \sum_{j=1}^{b} TP_j^r \tag{31}$$

4 Solution method

Here, two different structures of proposed supply chain which are integrated and non-integrated are considered to choose the best one from comparing their total profits with each other.

4.1 Integrated structure

In this section, an integrated three-level supply chain including multi-supplier, one manufacturer and several retailers is perceived which it, as a whole inventory system, faces the market demand thus $\sum_{k=1}^{n}\lambda_{ik}d_k^s = d_i^m = \sum_{j=1}^{b}d_{ji}^r = a - bS_i^r$; in turn the partners of the chain collaborate with each other to maximize the profit of the chain as a whole structure. The wide optimal approach as a centralized decision making process is considered as a solution method to model the behavior of its members. Under this approach, the chain has a decision maker who determines the optimal values of the decision variables where the quantity orders of suppliers and the selling prices of products are the decision variables. Optimizing the objective function of the chain is proved by showing its concavity as follows.

Theorem 1 *The objective function of supply chain, TP, is concave.*

Proof See "Appendix".

The roots of TP respect to q_k and S_i^r are the optimal values of q_k and S_i^r which are as below:

$$\begin{aligned} \frac{\partial^2 TP(q_k, S_i^r)}{\partial q_k} = &-\frac{h_k^s}{2} + \sum_{i=1}^{m}\frac{(a - bS_i^r)A_k^s}{\sum_{k=1}^{n}\lambda_{ik}q_k^2} - \sum_{i=1}^{m}\frac{h_i^m\sum_{k=1}^{n}\lambda_{ik}}{2}\left[1 - \frac{(1 + \alpha_i + \alpha_i^2)(a - bS_i^r)}{P_i}\right] \\ &+ \sum_{i=1}^{m}\frac{(a - bS_i^r)A_i^m}{\sum_{k=1}^{n}\lambda_{ik}q_k^2} - \sum_{j=1}^{b}\sum_{i=1}^{m}\frac{h_{ji}^r\rho_{ji}\sum_{k=1}^{n}\lambda_{ik}}{2}\left[1 - \frac{1}{\rho_{ji}}\right] + \sum_{j=1}^{b}\sum_{i=1}^{m}\frac{(a - bS_i^r)A_{ji}^r}{\rho_{ji}\sum_{k=1}^{n}\lambda_{ik}q_k^2} = 0 \end{aligned} \tag{32}$$

$$\frac{\partial^2 TP(q_k, S_i^r)}{\partial S_i^r} = -\frac{b\sum_{k=1}^n (S_k^s - C_k^s)}{\sum_{k=1}^n \lambda_{ik}} + \frac{b\sum_{k=1}^n A_k^s}{\sum_{k=1}^n \lambda_{ik} q_k} - b\left(S_i^r - C(P_i) - \sum_{k=1}^n S_k^d\right) + (a - bS_i^r)$$
$$- \frac{bh_i^m \sum_{k=1}^n \lambda_{ik} q_k}{2P_i}(1 + \alpha_i + \alpha_i^2) + \frac{bA_i^m}{\sum_{k=1}^n \lambda_{ik} q_k} + \sum_{j=1}^b \frac{bA_{ji}^r}{\rho_{ji}\sum_{k=1}^n \lambda_{ik} q_k} = 0 \tag{33}$$

Solving Eqs. (32) and (33), simultaneously, the optimal values of the integrated supply chain are obtained.

4.2 Non-integrated structure

In this section, whereas a non-integrated three-level supply chain is considered, so the members of chain do not collaborate with each other and each of them attempts to maximize one's profit. The Stackelberg game-theoretic approach as a decentralized decision making process is employed as a solution method among the members of the chain to model the relations among them where the non-competing suppliers and the manufacturer are followers and leaders, in the first stage, respectively and in the second stage, the manufacturer is follower and the non-competing retailers are leaders. In this approach, firstly, follower determines one's decision variables and then leader as the dominant of market optimizes own decision variables based on the best reaction of the follower (Leng and Parlar 2010; Seyed-Esfahani et al. 2011; Zhang and Liu 2013; He 2013; Taleizadeh and Noori-daryan 2014). In the first stage, we consider the suppliers as followers offering the delivering quantities of raw material to the manufacturer. Then the manufacturer and the retailers as leaders determine their own selling prices for each product. In order to optimize the objective function of supply chain network, we need to prove concavity of each member's objective function studied in the following.

Theorem 2 *The objective function of kth supplier, TP_k^d, is concave.*

Proof To proof the concavity of TP_k^d, the first and the second derivatives of objective function of kth supplier, shown in Eq. (6), respect to q_k are as below:

$$\frac{\partial TP_k^s(q_k)}{\partial q_k} = -\frac{h_k^s}{2} + \frac{d_k^s A_k^s}{q_k^2} = 0 \tag{34}$$

$$\frac{\partial^2 TP_k^s}{\partial^2 q_k} = -\frac{d_k^s A_k^s}{q_k^3} < 0 \tag{35}$$

The second derivative is strictly negative therefore the objective function of kth supplier is concave. The root of Eq. (34) is the optimal value of q_k which is:

$$q_k^* = \sqrt{\frac{2d_k^s A_k^s}{h_k^s}} \tag{36}$$

Theorem 3 *The objective function of the manufacturer, TP^m, is concave.*

Proof In order to prove concavity of manufacturer's objective function, the first and the second derivatives of the manufacturer's objective function, shown in Eq. (20), respect to S_i^m are as below:

$$\begin{aligned}\frac{\partial TP^m(S_i^m, q_k^*)}{\partial S_i^m} &= a - 2bS_i^m + bC(P_i) + b\sum_{k=1}^{n} S_k^s \\ &\quad - \frac{bh_m^i(1+\alpha_i+\alpha_i^2)\sum_{k=1}^{n}\lambda_{ik}q_k}{2P_i} + \frac{bA_i^m}{\sum_{k=1}^{n}\lambda_{ik}q_k} = 0\end{aligned} \tag{37}$$

$$\frac{\partial^2 TP^m(S_i^m, q_k^*)}{\partial S_i^{m2}} = -2b < 0 \tag{38}$$

Since the second derivative is strictly negative, the manufacturer's objective function is concave too. Similarly, the root of Eq. (37) is the optimal value of S_i^m which is:

$$\begin{aligned}S_i^{m*} &= \frac{a}{2b} + \frac{C(P_i)}{2} + \frac{\sum_{k=1}^{n} S_k^s}{2} - \frac{h_m^i(1+\alpha_i+\alpha_i^2)\sum_{k=1}^{n}\lambda_{ik}q_k^*}{4P_i} + \frac{A_i^m}{2\sum_{k=1}^{n}\lambda_{ik}q_k^*} \\ &= \frac{a}{2b} + \frac{C(P_i)}{2} + \frac{\sum_{k=1}^{n} S_k^s}{2} - \frac{h_m^i(1+\alpha_i+\alpha_i^2)\sum_{k=1}^{n}\lambda_{ik}\sqrt{\frac{2d_k^s A_k^s}{h_k^s}}}{4P_i} + \frac{A_i^m}{2\sum_{k=1}^{n}\lambda_{ik}\sqrt{\frac{2d_k^s A_k^s}{h_k^s}}} = 0\end{aligned} \tag{39}$$

Theorem 4 *The objective function of* jth *retailer, shown in Eq.* (30), *is concave.*

Proof Like previous cases, we have:

$$\frac{\partial TP_j^r(S_{ji}^r, S_i^{m*}, q_k^*)}{\partial S_{ji}^r} = a - 2bS_{ji}^r + bS_i^{m*} - \frac{bh_{ji}^r\sum_{k=1}^{n}\lambda_{ik}q_k^*}{2(a - bS_i^{m*})} + \frac{bA_{ji}^r}{\rho_{ji}\sum_{k=1}^{n}\lambda_{ik}q_k^*} = 0 \tag{40}$$

$$\frac{\partial^2 TP_j^r(S_{ji}^r, S_i^{m*}, q_k^*)}{\partial S_{ji}^{r2}} = -2b < 0 \tag{41}$$

According to Eq. (41), concavity of jth retailer's objective function is proven and the optimal value of S_{ji}^{r} using Eq. (40) and replacing $q_k^*$with $\sqrt{\frac{2d_k^s A_k^s}{h_k^s}}$ is:

$$
\begin{aligned}
S_{ji}^{r*} &= \frac{a}{2b} + \frac{S_i^{m*}}{2} - \frac{h_{ji}^{r} \sum_{k=1}^{n} \lambda_{ik} q_k^*}{4(a - bS_i^{m*})} + \frac{A_{ji}^{r}}{2\rho_{ji} \sum_{k=1}^{n} \lambda_{ik} q_k^*} \\
&= \frac{a}{2b} + \frac{S_i^{m*}}{2} - \frac{bh_{ji}^{r} \sum_{k=1}^{n} \lambda_{ik} \sqrt{\frac{2d_k^s A_k^s}{h_k^s}}}{2(a - bS_i^{m*})} + \frac{bA_{ji}^{r}}{\rho_{ji} \sum_{k=1}^{n} \lambda_{ik} \sqrt{\frac{2d_k^s A_k^s}{h_k^s}}}
\end{aligned} \tag{42}
$$

5 Real case study

5.1 Numerical example

In this section, a numerical example for proposed multi-product economic production quantity (EPQ) model in a three layers supply chain with rework process for two different integrated and Non-integrated structures is presented. According to our real case study which is in a pharmacological industry, we consider seven suppliers, one manufacturer and three retailers as the members of studied chain. Since each supplier supplies one kind of raw material, the number of suppliers is equal to the number of raw material (seven types of raw material) and manufacturer combines fixed percentage of the various kinds of them to produce seven products and sends them to the retailers. According to assumptions, the defective items generated during the production process, will be reworked after finishing production process at a same production rate and proportion of defective items for each product is constant. The raw materials are seven chemical materials including *Dibasic Calcium Phosphate*, *Sodium Benzoate*, *Gelatin Capsule*, *Guar Gum*, *Crystal Sugar, Flavor* and *Color* used to produce seven antibiotics tablets which are *Cefexime, Cephalexine Sodium*, *Amoxiclov 156, Amoxiclov 228, Amoxiclov 312, Amoxiclov 457, Co-Amoxiclov*. In this example, we consider $a = 10{,}000$, $b = 45$ and the related data of suppliers, manufacturer and retailers are shown in Tables 1, 2, and 3, respectively. The results of suppliers as the partners of

Table 1 The general data of suppliers

Distributers k	Raw material	C_k^s	h_k^s	S_k^s	d_k^s	A_k^s
1	*Dibasic Calsium Phosphate*	8	0.50	18	550	2
2	*Sodium Benzoate*	7	0.30	16	680	3
3	*Gelatine Capsule*	11	0.60	22	450	5
4	*Guar Gum*	9	0.55	20	500	4
5	*Crystal Sugar*	8	0.40	17	600	1.5
6	*Flavor*	7.5	0.35	21	650	4.5
7	*Color*	10	0.45	19	570	5.5

Table 2 The general data of manufacturer

Products	λ_{i1}	λ_{i2}	λ_{i3}	λ_{i4}	λ_{i5}	λ_{i6}	λ_{i7}	α_i	F_i	v_i	h_i^m	A_i^m
Cefexime	0.10	0.20	0.05	0.30	0.10	0.25	0.35	0.10	4000	0.01	0.9	30
Cephalexine Sodium	0.20	0.15	0.10	0.05	0.10	0.10	0.15	0.20	4800	0.02	1	35
Amoxiclov 156	0.30	0.20	0.25	0.05	0.10	0.05	0.05	0.25	4500	0.01	0.8	33
Amoxiclov 228	0.15	0.10	0.10	0.20	0.20	0.10	0.15	0.15	4200	0.015	0.7	37
Amoxiclov 312	0.10	0.05	0.20	0.10	0.20	0.10	0.05	0.10	4100	0.02	0.6	34
Amoxiclov 457	0.10	0.15	0.20	0.10	0.10	0.30	0.10	0.30	4300	0.01	0.8	36
Co-Amoxiclov	0.05	0.15	0.10	0.20	0.20	0.10	0.15	0.20	4700	0.015	0.7	33

Table 3 The general data of retailers

Products	ρ_{1i}	ρ_{2i}	ρ_{3i}	h_{1i}^r	h_{2i}^r	h_{3i}^r	A_{1i}^r	A_{2i}^r	A_{3i}^r
Cefexime	0.30	0.40	0.30	1.00	1.09	0.98	40	46	41
Cephalexine Sodium	0.20	0.50	0.30	1.05	1.00	1.06	47	49	48
Amoxiclov 156	0.35	0.30	0.35	1.10	1.04	1.19	43	47	46
Amoxiclov 228	0.10	0.30	0.60	1.20	0.99	1.10	42	45	48
Amoxiclov 312	0.25	0.30	0.45	1.02	1.01	1.24	45	40	44
Amoxiclov 457	0.50	0.15	0.35	1.15	1.13	1.03	41	43	40
Co-Amoxiclov	0.40	0.25	0.35	1.20	1.07	1.09	44	45	49

Table 4 The results of example

Distributers k	Raw material	Integrated structure q_k^*	Non-integrated structure q_k^*
1	*Dibasic Calsium Phosphate*	69.35	66.33
2	*Sodium Benzoate*	120.10	116.61
3	*Gelatine Capsule*	91.40	86.60
4	*Guar Gum*	88.70	85.28
5	*Crystal Sugar*	72.12	67.08
6	*Flavor*	133.75	129.28
7	*Color*	123.05	118.03

integrated structure and as the followers of non-integrated structure are given in Table 4 and the results of manufacturer and the retailers as the partners and the leaders of integrated and non-integrated structures, in turn, are presented in Table 5.

Based on the results, the total profit of supply chain under integrated structure is more about 35 % than the other one (i.e. non-integrated supply chain) because of lower selling price and more quantity orders than non-integrated chain.

Table 5 The results of example

Products	Integrated structure		Non-integrated structure				
	S_i^{r*}	TP	S_i^{m*}	S_{1i}^{r*}	S_{2i}^{r*}	S_{3i}^{r*}	TP
Cefexime	199.08		184.14	203.63	203.57	203.65	
Cephalexine Sodium	202.02		187.57	206.33	205.49	205.87	
Amoxiclov 156	201.21		184.57	204.08	204.27	204.12	
Amoxiclov 228	200.50	94,234.02	185.73	206.26	204.78	204.40	69,456
Amoxiclov 312	201.98		186.88	205.81	205.48	205.23	
Amoxiclov 457	200.31		184.30	203.62	204.58	203.78	
Co-Amoxiclov	202.09		186.15	204.77	205.16	204.94	

Table 6 Effects of the changes of followers' cost on the selling price of manufacturer

Parameters	% Changes					
	−75	−50	−25	+25	+50	+75
C_k^s						
S_1^{m*}	−0.053	−0.027	−0.011	+0.009	+0.017	+0.024
S_2^{m*}	−0.084	−0.045	−0.019	+0.016	+0.030	+0.043
S_3^{m*}	−0.073	−0.039	−0.017	+0.014	+0.026	+0.038
S_4^{m*}	−0.072	−0.039	−0.017	+0.014	+0.027	+0.039
S_5^{m*}	−0.078	−0.043	−0.019	+0.016	+0.031	+0.045
S_6^{m*}	−0.072	−0.037	−0.016	+0.013	+0.025	+0.035
S_7^{m*}	−0.068	−0.036	−0.016	+0.013	+0.025	+0.036
A_k^s						
S_1^{m*}	+0.070	+0.031	+0.012	−0.009	−0.016	−0.022
S_2^{m*}	+0.128	+0.055	+0.021	−0.015	−0.027	−0.036
S_3^{m*}	+0.112	+0.048	+0.018	−0.013	−0.023	−0.032
S_4^{m*}	+0.117	+0.050	+0.019	−0.013	−0.024	−0.032
S_5^{m*}	+0.136	+0.057	+0.021	−0.015	−0.026	−0.036
S_6^{m*}	+0.104	+0.045	+0.017	−0.012	−0.022	−0.030
S_7^{m*}	+0.107	+0.046	+0.017	−0.012	−0.022	−0.030

5.2 Sensitivity analysis on the decision variables of supply chain

Now, effects of the changes of the followers' costs on the decision variables of leaders are studied where ordering and purchasing costs are considered as the costs of followers and a sensitivity analysis is performed by increasing and decreasing parameters, at a time, by 25, 50 and 75 %. According to Table 6 showing the effects of the changes of followers' cost on the manufacturer's selling price as a leader. His

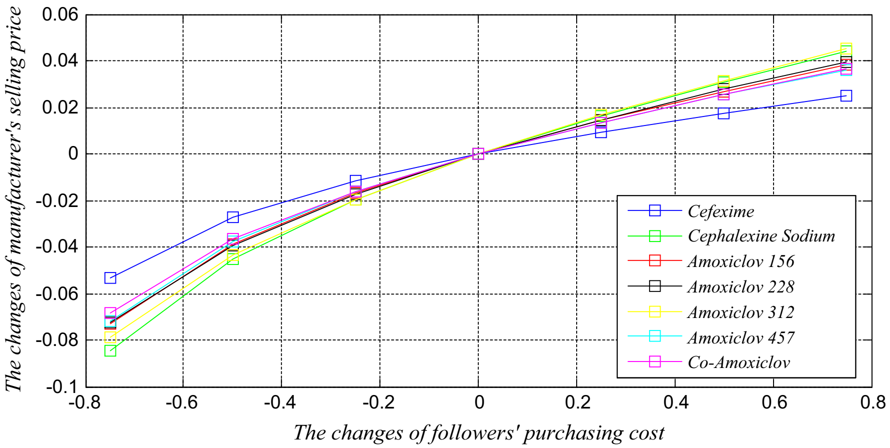

Fig. 5 Digram of the changes of manufacturer's selling price versus the changes of the followers' purchasing cost

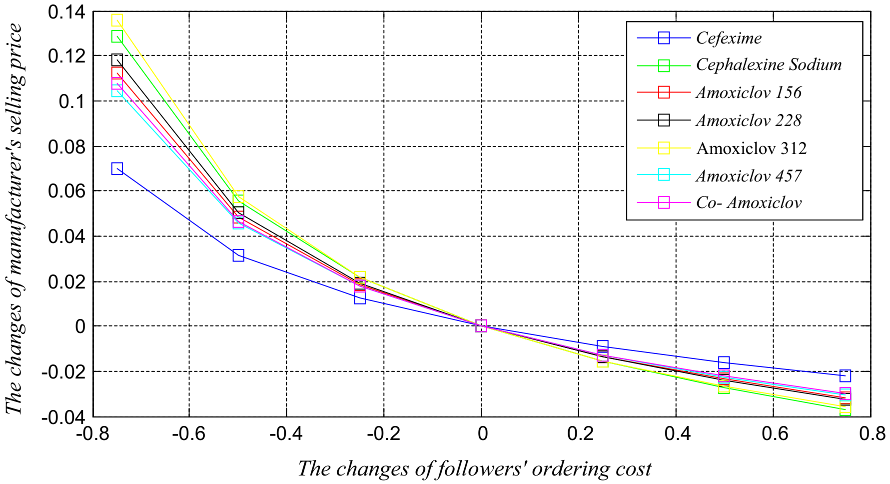

Fig. 6 Diagram of the changes of manufacturer's selling price versus the changes of the followers' ordering cost

selling price for each product is decreased by decreasing followers' purchasing cost and increasing their ordering cost.

Diagrams of the changes of manufacturer's selling price versus the changes of followers' costs are presented in Figs. 5 and 6 (Table 6).

Therefore, retailers' selling price is increased by increasing followers' purchasing cost and decreasing their ordering cost (Tables 7, 8, 9).

Table 7 Effects of the changes of followers' cost on the selling price of the first retailer

Parameters	% Changes					
	−75	−50	−25	+25	+50	+75
C_k^s						
S_{11}^{r*}	−0.126	−0.072	−0.032	+0.028	+0.054	+0.077
S_{12}^{r*}	−0.358	−0.208	−0.095	+0.083	+0.159	+0.228
S_{13}^{r*}	−0.177	−0.102	−0.046	+0.041	+0.078	+0.111
S_{14}^{r*}	−0.565	−0.329	−0.150	+0.132	+0.251	+0.361
S_{15}^{r*}	−0.315	−0.183	−0.083	+0.073	+0.140	+0.201
S_{16}^{r*}	−0.102	−0.058	−0.026	+0.023	+0.043	+0.062
S_{17}^{r*}	−0.156	−0.090	−0.041	+0.035	+0.068	+0.097
A_k^s						
S_{11}^{r*}	+0.238	+0.099	+0.037	−0.025	−0.045	−0.060
S_{12}^{r*}	+0.706	+0.293	+0.109	−0.075	−0.130	−0.173
S_{13}^{r*}	+0.344	+0.143	+0.053	−0.036	−0.064	−0.085
S_{14}^{r*}	+1.118	+0.463	+0.173	−0.118	−0.206	−0.274
S_{15}^{r*}	+0.622	+0.258	+0.096	−0.066	−0.115	−0.153
S_{16}^{r*}	+0.192	+0.080	+0.030	−0.020	−0.036	−0.048
S_{17}^{r*}	+0.300	+0.125	+0.047	−0.032	−0.056	−0.075

Table 8 Effects of the changes of followers' cost on the selling price of the second retailer

Parameters	% Changes					
	−75	−50	−25	+25	+50	+75
C_k^s						
S_{21}^{r*}	−0.111	−0.063	−0.028	+0.024	+0.047	+0.067
S_{22}^{r*}	−0.153	−0.088	−0.040	+0.035	+0.067	+0.096
S_{23}^{r*}	−0.224	−0.130	−0.059	+0.052	+0.098	+0.142
S_{24}^{r*}	−0.207	−0.120	−0.054	+0.048	+0.091	+0.131
S_{25}^{r*}	−0.235	−0.136	−0.062	+0.054	+0.104	+0.149
S_{26}^{r*}	−0.335	−0.194	−0.088	+0.078	+0.148	+0.213
S_{27}^{r*}	−0.249	−0.144	−0.066	+0.057	+0.110	+0.158
A_k^s						
S_{21}^{r*}	+0.206	+0.086	+0.032	−0.022	−0.039	−0.052
S_{22}^{r*}	+0.297	+0.123	+0.046	−0.031	−0.055	−0.073
S_{23}^{r*}	+0.438	+0.182	+0.068	−0.046	−0.081	−0.108
S_{24}^{r*}	+0.404	+0.168	+0.062	−0.043	−0.075	−0.100
S_{25}^{r*}	+0.462	+0.192	+0.071	−0.049	−0.085	−0.114
S_{26}^{r*}	+0.657	+0.273	+0.102	−0.070	−0.121	−0.162
S_{27}^{r*}	+0.487	+0.202	+0.075	−0.052	−0.090	−0.120

Table 9 Effects of the changes of followers' cost on the selling price of the third retailer

Parameters	% Changes					
	−75	−50	−25	+25	+50	+75
C_k^s						
S_{31}^{r*}	−0.129	−0.074	−0.033	+0.029	+0.055	+0.079
S_{32}^{r*}	−0.246	−0.143	−0.065	+0.057	+0.109	+0.156
S_{33}^{r*}	−0.190	−0.110	−0.050	+0.043	+0.083	+0.119
S_{34}^{r*}	−0.114	−0.065	−0.029	+0.026	+0.049	+0.071
S_{35}^{r*}	−0.175	−0.101	−0.046	+0.040	+0.077	+0.110
S_{36}^{r*}	−0.138	−0.079	−0.036	+0.031	+0.060	+0.086
S_{37}^{r*}	−0.196	−0.113	−0.051	+0.045	+0.086	+0.123
A_k^s						
S_{31}^{r*}	+0.244	+0.102	+0.038	−0.026	−0.046	−0.061
S_{32}^{r*}	+0.482	+0.200	+0.075	−0.051	−0.089	−0.119
S_{33}^{r*}	+0.368	+0.153	+0.057	−0.039	−0.068	−0.091
S_{34}^{r*}	+0.217	+0.090	+0.034	−0.023	−0.041	−0.054
S_{35}^{r*}	+0.341	+0.141	+0.053	−0.036	−0.063	−0.084
S_{36}^{r*}	+0.265	+0.110	+0.041	−0.028	−0.049	−0.066
S_{37}^{r*}	+0.380	+0.158	+0.059	−0.040	−0.070	−0.094

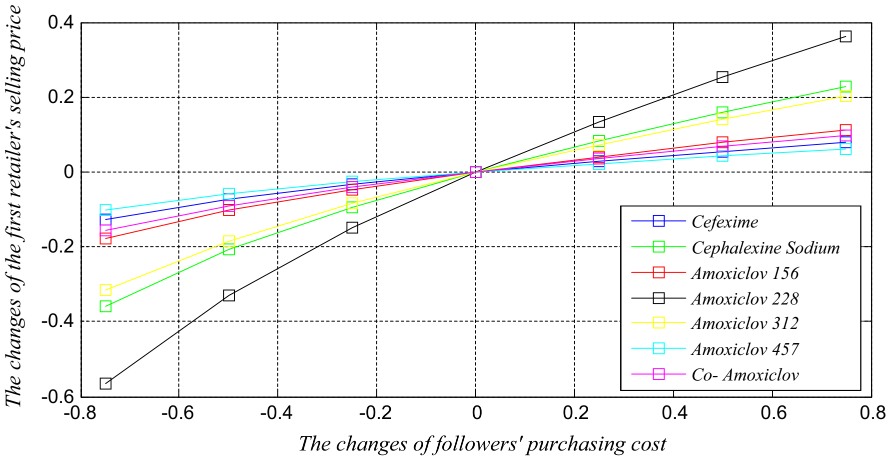

Fig. 7 Diagram of the changes of the first retailer's selling price versus the changes of the followers' purchasing cost

Finally, Figs. 7 and 8 demonstrate diagram of the changes of the first retailer's selling price versus the changes of followers' purchasing and ordering costs and diagrams of the changes of the second and the third retailer's selling price versus the

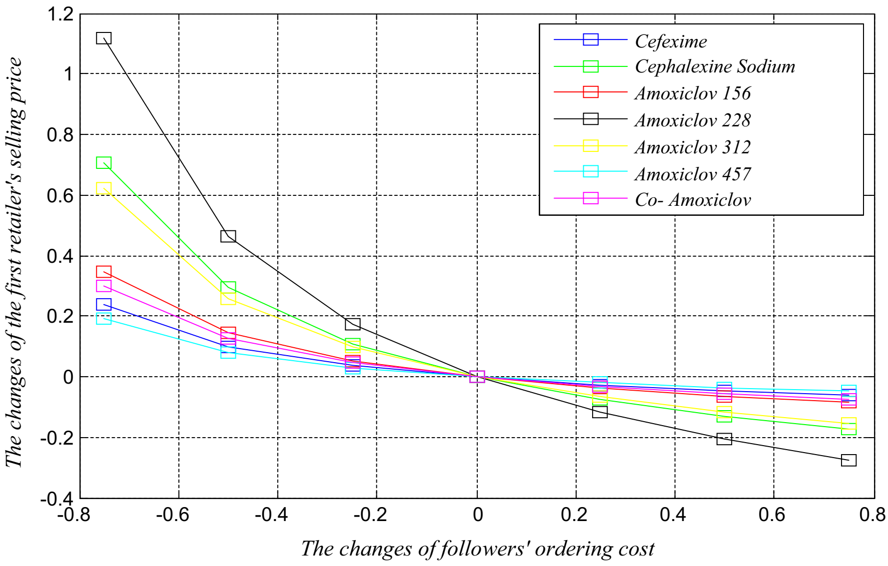

Fig. 8 Diagram of the changes of the first retailer's selling price versus the changes of the followers' ordering cost

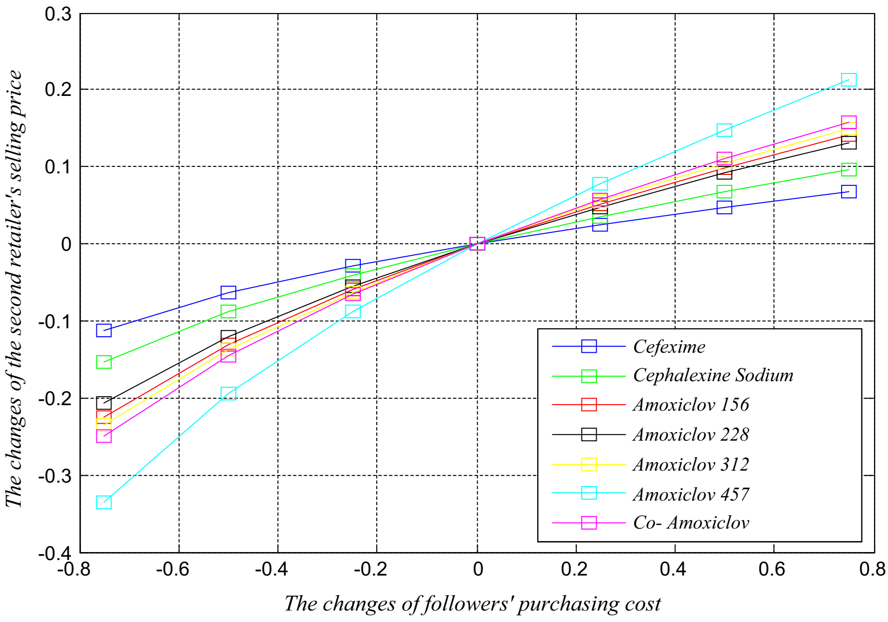

Fig. 9 Diagram of the changes of the second retailer's selling price versus the changes of the followers' purchasing cost

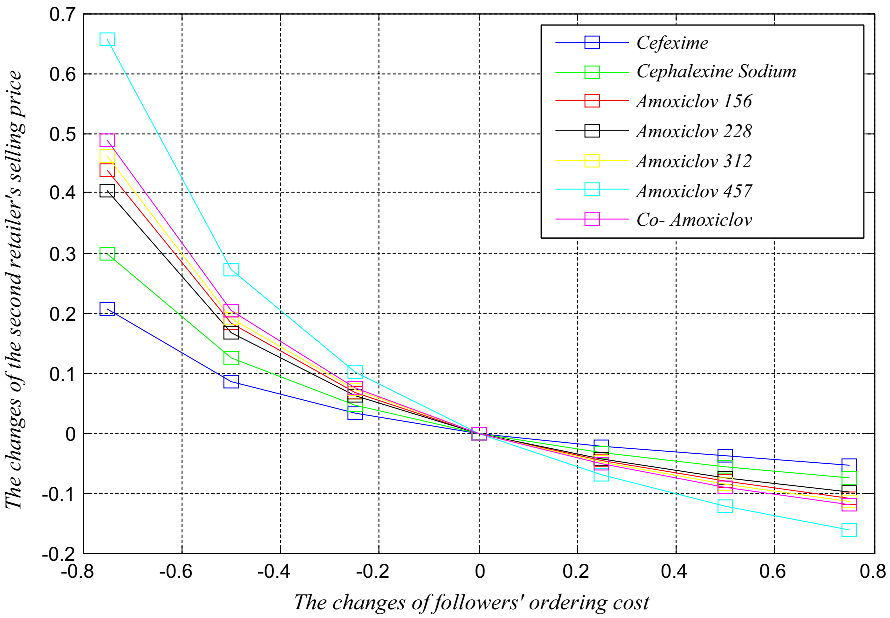

Fig. 10 Diagram of the changes of the second retailer's selling price versus the changes of the followers' ordering cost

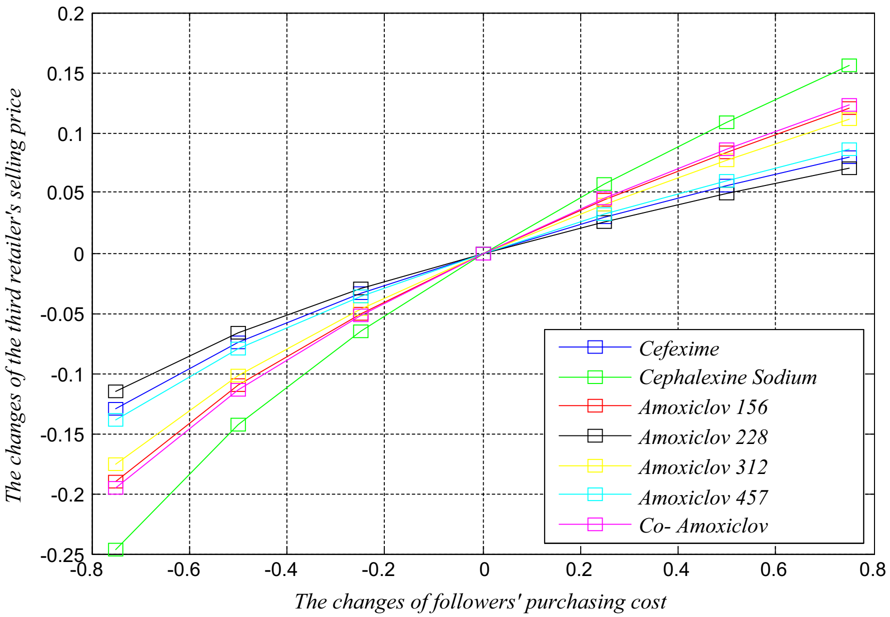

Fig. 11 Diagram of the changes of the third retailer's selling price versus the changes of the followers' purchasing cost

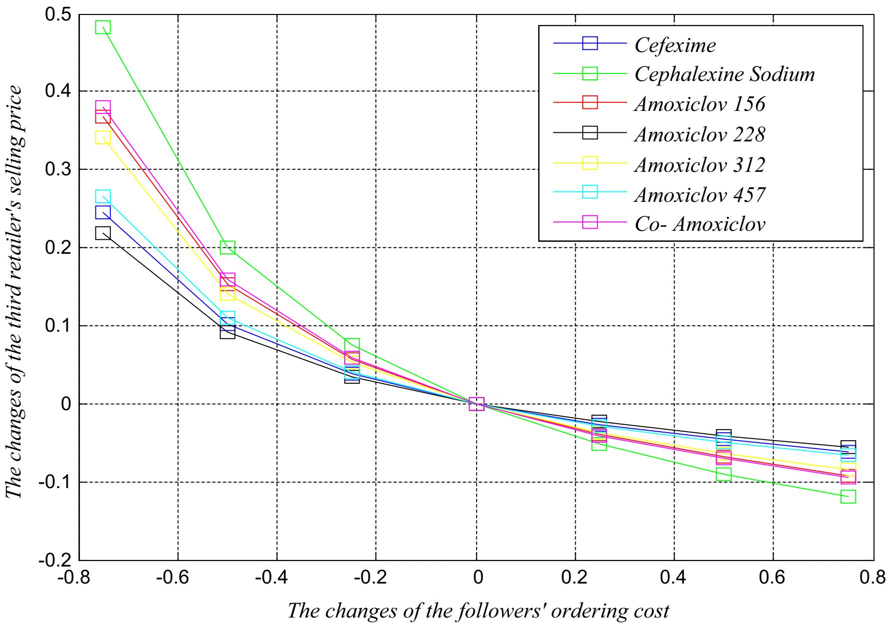

Fig. 12 Diagram of the changes of the third retailer's selling price versus the changes of followers' ordering cost

changes of followers' costs are shown in Figs. 9, 10, 11 and 12, respectively (Tables 7, 8, 9).

6 Conclusion

In this research, we developed a multi-product economic production quantity (EPQ) model with rework process in a three levels supply chain including multiple non-competing suppliers, one manufacturer and multiple non-competing retailers under both integrated and non-integrated structures. Each supplier supplies one type of raw material to the manufacturer and he produces products by combination of certain percentage of various kinds of raw material. During the production process, the defective products are generated which will be reworked at a same production rate after finishing the regular production process. Then the manufacturer satisfies demand of the retailers. In this model, we consider a wide optimal and Stackelberg game-theoretic approaches among the partners of the chain for integrated and non-integrated chains, respectively. Under the wide optimal approach, all the partners cooperate with each other and attempt to improve the chain profit, so a decision maker decides about the optimal values of decision variables of supply chain while under the Stackelberg approach, the suppliers are the followers and the manufacturer and the retailers are the leaders such that each members of the chain tries to enhance own profit. The aim of this research is to determine the quantity order of the suppliers, the manufacturer's and the retailers' selling prices. We presented that the obtained profit objective functions are concave and the closed form solutions are derived for the optimal values of decision variables. Finally, a numerical example is presented for illustrating the introduced model and some sensitivity analyses are performed on the decision variables of non-integrated supply chain. The results show that the profit of integrated supply chain is more about 35 % than the profit of non-integrated chain. For future research topics, we suggest developing this study by adding backordering for all levels of the chain or defining demand rate as a random variable.

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Appendix: Proofing the concavity of $TP(q_k, S_i^r)$

$TP(q_k, S_i^r)$ is concave when $X.H.X^T < 0$, where $X = [\,q_k \quad S_i^r\,]$,

$$X^T = \begin{bmatrix} q_k \\ S_i^r \end{bmatrix}, \quad H = \begin{bmatrix} \dfrac{\partial^2 TP(q_k, S_i^r)}{\partial q_k^2} & \dfrac{\partial^2 TP(q_k, S_i^r)}{\partial q_k \partial S_i^r} \\ \dfrac{\partial^2 TP(q_k, S_i^r)}{\partial S_i^r \partial q_k} & \dfrac{\partial^2 TP(q_k, S_i^r)}{\partial S_i^{r2}} \end{bmatrix} \text{ and}$$

$$\begin{aligned} TP(q_k,S_i^r) = {} & \sum_{k=1}^{n} \left(S_k^s - C_k^s\right)\left(a - bS_i^r\right) - \left(\frac{h_k^s q_k}{2} + \frac{(a-bS_i^r)A_k^s}{q_k}\right) \\ & + \sum_{i=1}^{m} \left[\left(\left(S_i^r - C(P_i) - \sum_{k=1}^{n} S_k^s\right)(a - bS_i^r)\right)\right. \\ & - \left(\frac{h_i^m \sum_{k=1}^{n} \lambda_{ik} q_k}{2}\left[1 - \frac{(1+\alpha_i+\alpha_i^2)(a-bS_i^r)}{P_i}\right] + \frac{(a-bS_i^r)A_i^m}{\sum_{k=1}^{n} \lambda_{ik} q_k}\right) \\ & + \sum_{j=1}^{b}\sum_{i=1}^{m} \left(S_i^r - S_i^r\right)(a - bS_i^r) - \left(\frac{h_{ji}^r \rho_{ji} \sum_{k=1}^{n} \lambda_{ik} q_k}{2}\left[1 - \frac{1}{\rho_{ji}}\right] + \frac{(a-bS_i^r)A_{ji}^r}{\rho_{ji}\sum_{k=1}^{n} \lambda_{ik} q_k}\right) \end{aligned}$$

Then, we have:

$$[\,q_k \quad S_i^r\,] \begin{bmatrix} \dfrac{\partial^2 TP(q_k, S_i^r)}{\partial q_k^2} & \dfrac{\partial^2 TP(q_k, S_i^r)}{\partial q_k \partial S_i^r} \\ \dfrac{\partial^2 TP(q_k, S_i^r)}{\partial S_i^r \partial q_k} & \dfrac{\partial^2 TP(q_k, S_i^r)}{\partial S_i^{r2}} \end{bmatrix} \begin{bmatrix} q_k \\ S_i^r \end{bmatrix} < 0 \tag{43}$$

where,

$$\frac{\partial^2 TP(q_k, S_i^r)}{\partial q_k^2} = -\sum_{i=1}^{m}\left[\frac{2(a-bS_i^r)A_k^s}{\sum_{k=1}^{n}\lambda_{ik}q_k^3} + \frac{2(a-bS_i^r)A_i^m}{\sum_{k=1}^{n}\lambda_{ik}q_k^3} + \sum_{j=1}^{b}\frac{2(a-bS_i^r)A_{ji}^r}{\rho_{ji}\sum_{k=1}^{n}\lambda_{ik}q_k^3}\right] < 0 \tag{44}$$

$$\begin{aligned}\frac{\partial^2 TP(q_k, S_i^r)}{\partial q_k \partial S_i^r} &= -\frac{bA_k^s}{\sum_{k=1}^{n}\lambda_{ik}q_k^2} - \frac{bh_i^m \sum_{k=1}^{n}\lambda_{ik}}{2P_i}(1+\alpha_i+\alpha_i^2) - \frac{bA_i^m}{\sum_{k=1}^{n}\lambda_{ik}q_k^2}\\ &\quad - \sum_{j=1}^{b}\frac{bA_{ji}^r}{\rho_{ji}\sum_{k=1}^{n}\lambda_{ik}q_k^2}\\ &= g < 0\end{aligned} \tag{45}$$

$$\begin{aligned}\frac{\partial^2 TP(q_k, S_i^r)}{\partial S_i^r \partial q_k} &= -\frac{bA_k^s}{\sum\limits_{k=1}^{n}\lambda_{ik}q_k^2} - \frac{bh_i^m \sum\limits_{k=1}^{n}\lambda_{ik}}{2P_i}(1+\alpha_i+\alpha_i^2) - \frac{bA_i^m}{\sum\limits_{k=1}^{n}\lambda_{ik}q_k^2} - \sum_{j=1}^{b}\frac{bA_{ji}^r}{\rho_{ji}\sum\limits_{k=1}^{n}\lambda_{ik}q_k^2}\\ &= g < 0\end{aligned} \tag{46}$$

$$\frac{\partial^2 TP(q_k, S_i^r)}{\partial S_i^{r2}} = -2b < 0 \tag{47}$$

Therefore, we have:

$$[\,q_k \quad S_i^r\,]\begin{bmatrix} -\sum\limits_{i=1}^{m}\left[\frac{2(a-bS_i^r)A_k^s}{\sum_{k=1}^{n}\lambda_{ik}q_k^3} + \frac{2(a-bS_i^r)A_i^m}{\sum_{k=1}^{n}\lambda_{ik}q_k^3} + \sum\limits_{j=1}^{b}\frac{2(a-bS_i^r)A_{ji}^r}{\rho_{ji}\sum_{k=1}^{n}\lambda_{ik}q_k^3}\right] & g \\ g & -2b \end{bmatrix}\begin{bmatrix} q_k \\ S_i^r \end{bmatrix}$$

$$= \begin{bmatrix} -\sum\limits_{i=1}^{m}\left[\frac{2(a-bS_i^r)A_k^s}{\sum_{k=1}^{n}\lambda_{ik}q_k^2} + \frac{2(a-bS_i^r)A_i^m}{\sum_{k=1}^{n}\lambda_{ik}q_k^2} + \sum\limits_{j=1}^{b}\frac{2(a-bS_i^r)A_{ji}^r}{\rho_{ji}\sum_{k=1}^{n}\lambda_{ik}q_k^2}\right] & \begin{array}{c} -\frac{bA_k^s}{\sum_{k=1}^{n}\lambda_{ik}q_k} - \frac{bh_i^m\sum_{k=1}^{n}\lambda_{ik}q_k}{2P_i}(1+\alpha_i+\alpha_i^2) \\ -\frac{bA_i^m}{\sum_{k=1}^{n}\lambda_{ik}q_k} - \frac{b\sum_{j=1}^{b}A_{ji}^r}{\sum_{j=1}^{b}\rho_{ji}\sum_{k=1}^{n}\lambda_{ik}q_k} \end{array} \\ \begin{array}{c} -\frac{bS_i^rA_k^s}{\sum_{k=1}^{n}\lambda_{ik}q_k^2} - \frac{bh_i^m S_i^r\sum_{k=1}^{n}\lambda_{ik}}{2P_i}(1+\alpha_i+\alpha_i^2) \\ -\frac{bS_i^rA_i^m}{\sum_{k=1}^{n}\lambda_{ik}q_k^2} - \frac{bS_i^r\sum_{j=1}^{b}A_{ji}^r}{\sum_{j=1}^{b}\rho_{ji}\sum_{k=1}^{n}\lambda_{ik}q_k^2} \end{array} & -2bS_i^r \end{bmatrix}\begin{bmatrix} q_k \\ S_i^r \end{bmatrix}$$

$$\begin{aligned}&= -\frac{2aA_k^s}{\sum_{k=1}^{n}\lambda_{ik}q_k} - \frac{2aA_i^m}{\sum_{k=1}^{n}\lambda_{ik}q_k} - \frac{2a\sum_{j=1}^{b}A_{ji}^r}{\sum_{k=1}^{n}\rho_{ji}\sum_{k=1}^{n}\lambda_{ik}q_k} - \frac{2bS_i^r h_i^m \sum_{k=1}^{n}\lambda_{ik}q_k}{2P_i}\\ &\quad (1+\alpha_i+\alpha_i^2) - 2bS_i^r < 0\end{aligned} \tag{48}$$

Furthermore, the concavity is obvious.

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